

9.2 Answers

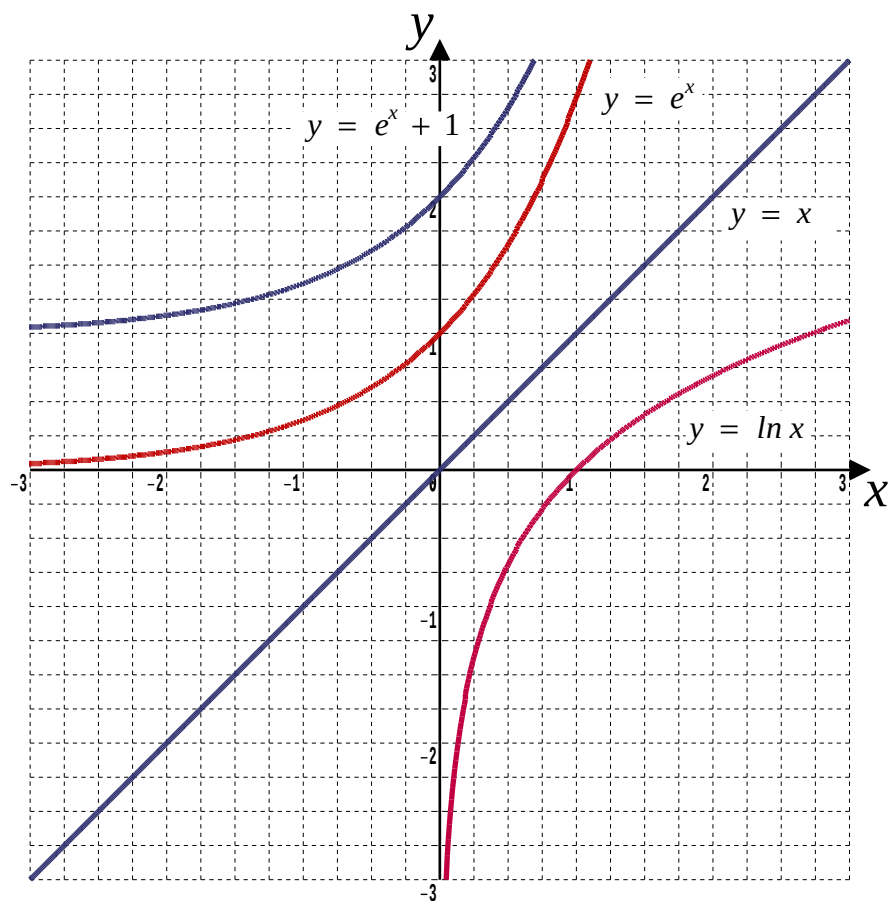
A-Level Pure Mathematics, Year 2 Functions II

Answer 1

(a) $f(x) \in \mathbb{R}, f(x) > 0$

[1 mark]

(b)



[3 marks]

Answer 2

$$f(x) = 64$$

$$e^{3 \ln x} = 64$$

$$e^{\ln x^3} = 64$$

$$x^3 = 64$$

$$x = 4$$

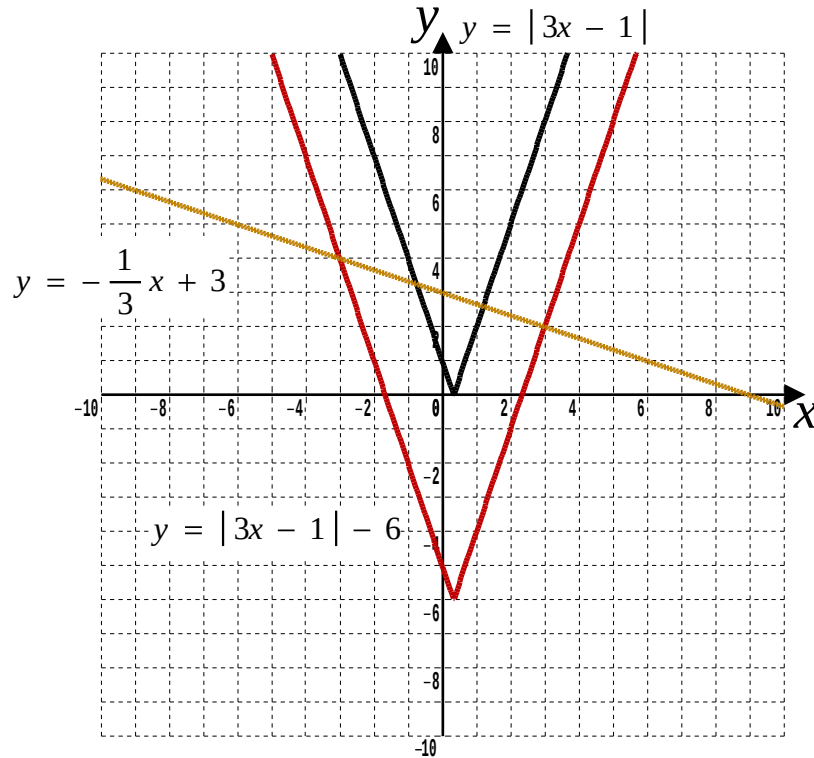
[2 marks]

Answer 3

(i) Black line on graph : $y = |3x - 1|$

It touches the x-axis at $\left(\frac{1}{3}, 0\right)$ and crosses the y-axis at $(0, 1)$

[3 marks]



(ii) The graph of $g(x)$ is $f(x)$ translated $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$ to give the red line on the graph

It crosses the y-axis at $(0, -5)$ and the x-axis when $|3x - 1| - 6 = 0$

That's when $+(3x - 1) = 6 \Rightarrow x = \frac{7}{3} \Rightarrow \left(\frac{7}{3}, 0\right)$

or when $-(3x - 1) = 6 \Rightarrow x = -\frac{5}{3} \Rightarrow \left(-\frac{5}{3}, 0\right)$

[4 marks]

(iii) $+(3x - 1) - 6 = -\frac{1}{3}x + 3$ $-(3x - 1) - 6 = -\frac{1}{3}x + 3$

$$9x - 21 = -x + 9$$

$$-9x - 15 = -x + 9$$

$$10x = 30$$

$$-8x = 24$$

$$x = 3 \Rightarrow (3, 2)$$

$$x = -3 \Rightarrow (-3, 4)$$

[6 marks]

(iv) Gold line of $y = -\frac{1}{3}x + 3$ added to graph

[1 mark]

Answer 4

$$\begin{aligned}
 \text{(i)} \quad fg(x) &= f(e^{3x} + 4) \\
 &= \ln(e^{3x} + 4 - 4) \\
 &= \ln(e^{3x}) \\
 &= 3x
 \end{aligned}$$

$$\text{Range } fg(x) \in \mathbb{R}$$

[3 marks]

$$\begin{aligned}
 \text{(ii)} \quad fg(x) &= 21 \\
 3x &= 21 \\
 x &= 7
 \end{aligned}$$

[1 mark]**Answer 5**

$$\begin{aligned}
 \text{(i)} \quad pq(x) &= p(\ln(x - 3)) \\
 &= e^{2\ln(x-3)} - 25 \\
 &= e^{\ln(x-3)^2} - 25 \\
 &= (x - 3)^2 - 25 \quad * * * \\
 &= x^2 - 6x + 9 - 25 \\
 &= x^2 - 6x - 16 \leftarrow \text{Full Marks}
 \end{aligned}$$

Completing the square to get the range

$$\begin{aligned}
 &= [x^2 - 6x] - 16 \\
 &= [(x - 3)^2 - 9] - 16 \\
 &= (x - 3)^2 - 25 \quad * * *
 \end{aligned}$$

 $\therefore$ The range is $pq(x) \in \mathbb{R}, pq(x) \geq -25$ **[5 marks]**

$$\text{(ii)} \quad pq(x) = 0$$

$$x^2 - 6x - 16 = 0$$

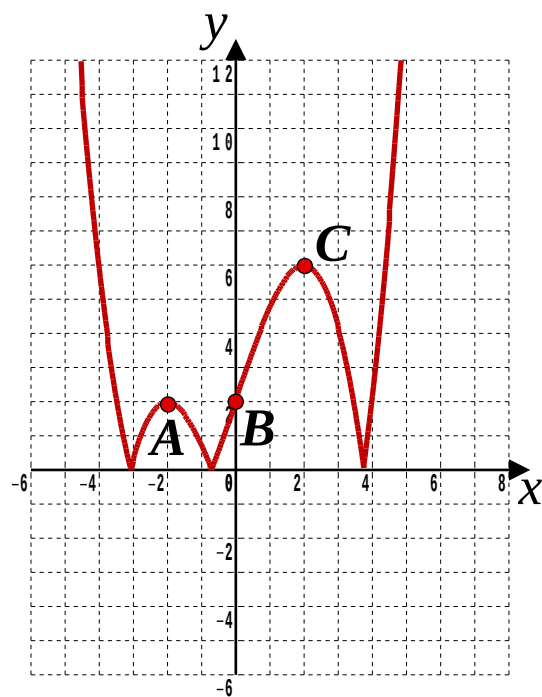
$$(x - 8)(x + 2) = 0$$

$$\text{Either } x = 8 \quad \text{or } x = -2$$

[2 marks]

Answer 6

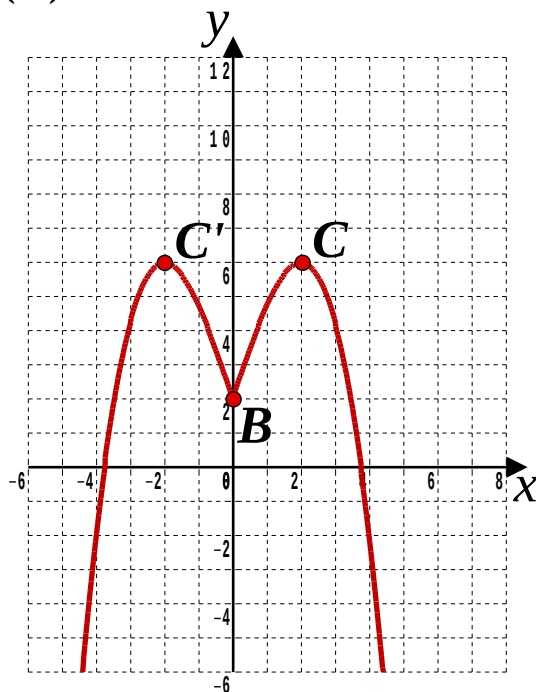
(i)



$A(-2, 2), B(0, 2), C(2, 6)$

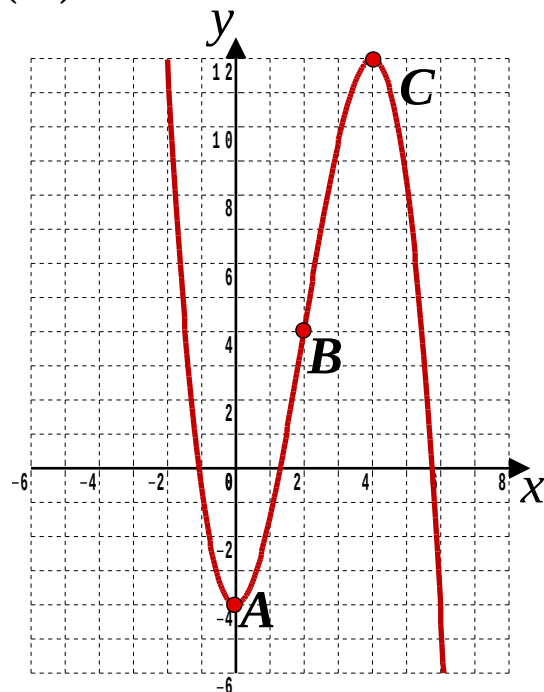
[3 marks]

(ii)



A is lost, $B(0, 2), C(2, 6), C'(-2, 6)$

(iii)



$A(0, -4), B(2, 4), C(4, 12)$

[3, 4 marks]

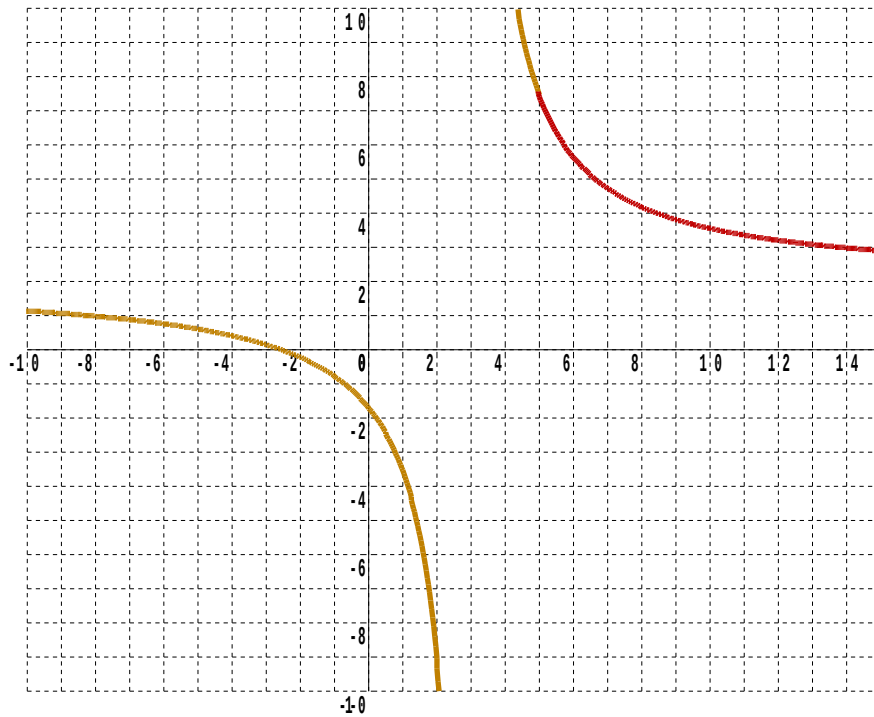
Answer 7

(a)

$$\begin{aligned}gg(5) &= g\left(\frac{2 \times 5 + 5}{5 - 3}\right) \\&= g\left(\frac{15}{2}\right) \\&= g(7.5) \\&= \frac{2 \times 7.5 + 5}{7.5 - 3} \\&= \frac{20}{4.5} \\&= \frac{40}{9}\end{aligned}$$

[2 marks]

(b)



The red line is the part of the function for which $x \geq 5$

There is a horizontal asymptote of $y = 2$

So the range is $2 < g(x) \leq 7.5$

[1 mark]

(c)

$$g(x) = \frac{2x + 5}{x - 3} \quad x \geq 5$$

$$y = \frac{2x + 5}{x - 3}$$

Swapping $x \Leftrightarrow y$ because input $\Leftrightarrow$ output

$$x = \frac{2y + 5}{y - 3}$$

$$x(y - 3) = 2y + 5$$

$$xy - 3x = 2y + 5$$

$$xy - 2y = 3x + 5$$

$$y(x - 2) = 3x + 5$$

$$y = \frac{3x + 5}{x - 2}$$

$$g^{-1}(x) = \frac{3x + 5}{x - 2}$$

The domain of the inverse function is the range of the original function

So the domain is $2 < x \leq 7.5$

[3 marks]

Answer 8

$$f^{-1}(x) = \frac{2x - 5}{x}$$

$$y = \frac{2x - 5}{x}$$

$$x \Leftrightarrow y$$

$$x = \frac{2y - 5}{y}$$

$$xy = 2y - 5$$

$$2y - xy = 5$$

$$y(2 - x) = 5$$

$$y = \frac{5}{2 - x}$$

$$f(x) = \frac{5}{2 - x} \quad x \neq 2$$

(a)

$$fg(x) = f(\sqrt{2x - k})$$

$$= \frac{5}{2 - \sqrt{2x - k}} \quad x \geq \frac{k}{2}, \quad x \neq 2 + \frac{k}{2}$$

[3 marks]**(b)**

$$gg(10) = 2$$

$$g(\sqrt{20 - k}) = 2$$

$$\sqrt{2\sqrt{20 - k} - k} = 2$$

$$2\sqrt{20 - k} - k = 4$$

$$2\sqrt{20 - k} = 4 + k$$

$$4(20 - k) = 16 + 8k + k^2$$

$$k^2 + 12k - 64 = 0$$

$$(k + 16)(k - 4) = 0$$

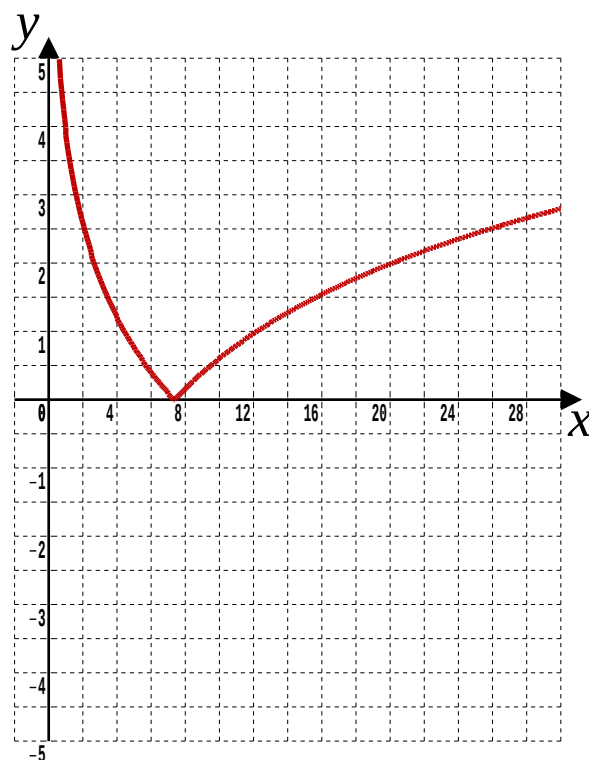
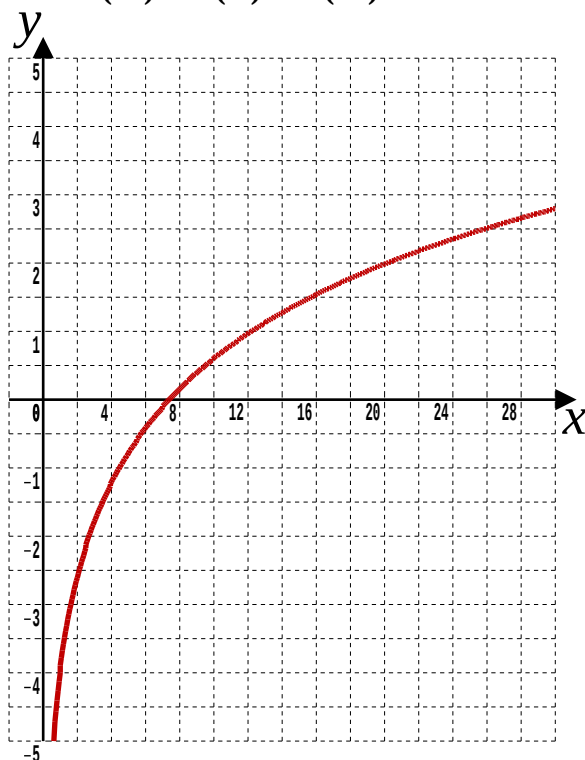
Either $k = -16$ (Reject as told k is positive)

$$\text{Or } k = 4$$

[3 marks]

Answer 9

(a) (i) and (ii)



x-axis contact, $2 \ln(x) - 4 = 0 \Rightarrow \ln(x) = 2 \Rightarrow x = e^2$ (About 7.39)

Asymptote on both graphs is the y-axis which has equation $x = 0$

[5 marks]

(b) $| f(x) | = 4$

$$2 \ln(x) - 4 = 4$$

$$\ln(x) = 4$$

$$x = e^4 \text{ (About 54.6)}$$

$$-2 \ln(x) + 4 = 4$$

$$\ln(x) = 0$$

$$x = 1$$

[4 marks]

(c) $gf(x) = g(2 \ln(x) - 4)$

$$= e^{2 \ln x - 4 + 5} - 2$$

$$= e^{\ln x^2 + 1} - 2$$

$$= e x^2 - 2$$

[3 marks]

(d) $gf(x) > -2$

[1 mark]

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