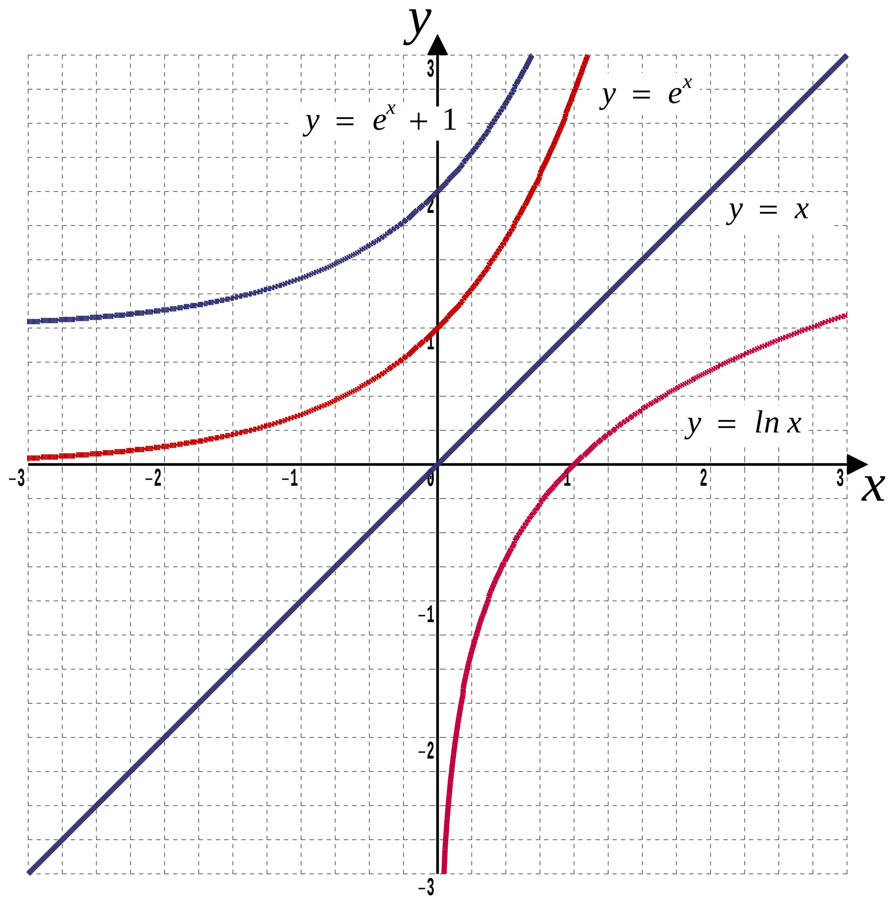
Answer 1

(a) $f(x) \in \mathbb{R},\ f(x) > 0$

[1 mark]

(b)



[3 marks]

Answer 2

$$f(x) = 64$$
$$e^{3\ln x} = 64$$
$$e^{\ln x^3} = 64$$
$$x^3 = 64$$
$$x = 4$$

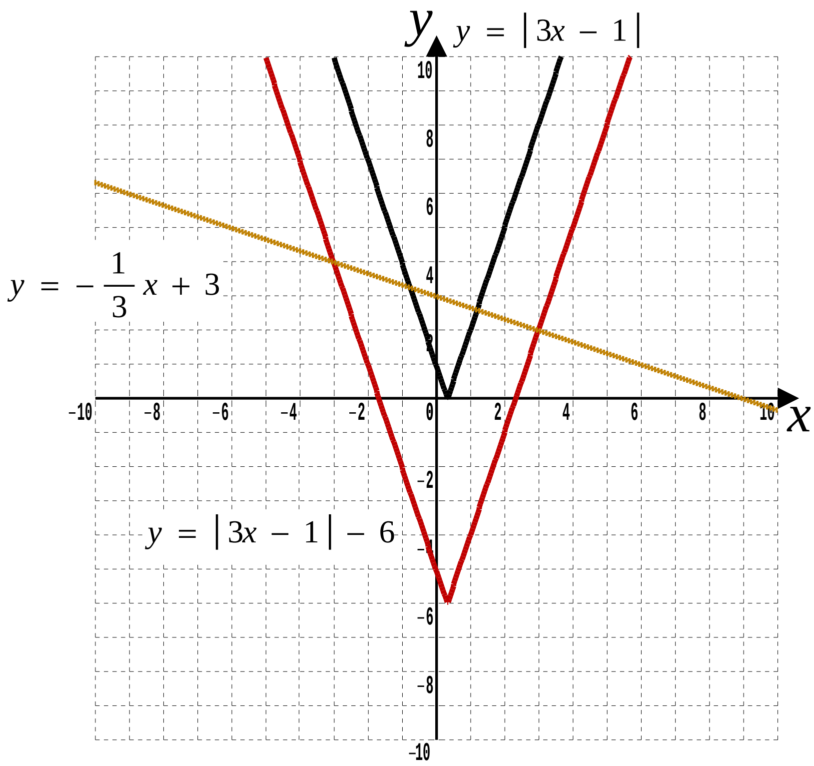
[2 marks]

Answer 3

(i) Black line on graph : $y = |3x - 1|$

It touches the x-axis at $\left(\frac{1}{3},\ 0\right)$ and crosses the y-axis at $(0,\ 1)$

[3 marks]



(ii) The graph of $g(x)$ is $f(x)$ translated $\begin{pmatrix} 0 \\ -6 \end{pmatrix}$ to give the red line on the graph

It crosses the y-axis at $(0, -5)$ and the x-axis when $|3x - 1| - 6 = 0$

That's when $+(3x - 1) = 6 \Rightarrow x = \frac{7}{3} \Rightarrow \left(\frac{7}{3},\ 0\right)$

or when $-(3x - 1) = 6 \Rightarrow x = -\frac{5}{3} \Rightarrow \left(-\frac{5}{3},\ 0\right)$

[4 marks]

(iii)

$$+(3x - 1) - 6 = -\frac{1}{3}x + 3 \qquad -(3x - 1) - 6 = -\frac{1}{3}x + 3$$

$$9x - 21 = -x + 9 \qquad -9x - 15 = -x + 9$$

$$10x = 30 \qquad -8x = 24$$

$$x = 3 \Rightarrow (3,\ 2) \qquad x = -3 \Rightarrow (-3,\ 4)$$

[6 marks]

(iv) Gold line of $y = -\frac{1}{3}x + 3$ added to graph

[1 mark]

Answer 4

(i)

$$fg(x) = f\left(e^{3x} + 4\right)$$
$$= ln\left(e^{3x} + 4 - 4\right)$$
$$= ln\left(e^{3x}\right)$$
$$= 3x$$

Range $fg(x) \in \mathbb{R}$

[3 marks]

(ii)

$$fg(x) = 21$$
$$3x = 21$$
$$x = 7$$

[1 mark]

Answer 5

(i)

$$pq(x) = p\left(ln(x - 3)\right)$$
$$= e^{2\,ln(x-3)} - 25$$
$$= e^{ln(x-3)^2} - 25$$
$$= (x - 3)^2 - 25 \quad * * *$$
$$= x^2 - 6x + 9 - 25$$
$$= x^2 - 6x - 16 \leftarrow \text{Full Marks}$$

Completing the square to get the range

$$= \left[x^2 - 6x\right] - 16$$
$$= \left[(x - 3)^2 - 9\right] - 16$$
$$= (x - 3)^2 - 25 \quad * * *$$

$\therefore$ The range is $pq(x) \in \mathbb{R},\ pq(x) \geqslant -25$

[5 marks]

(ii)

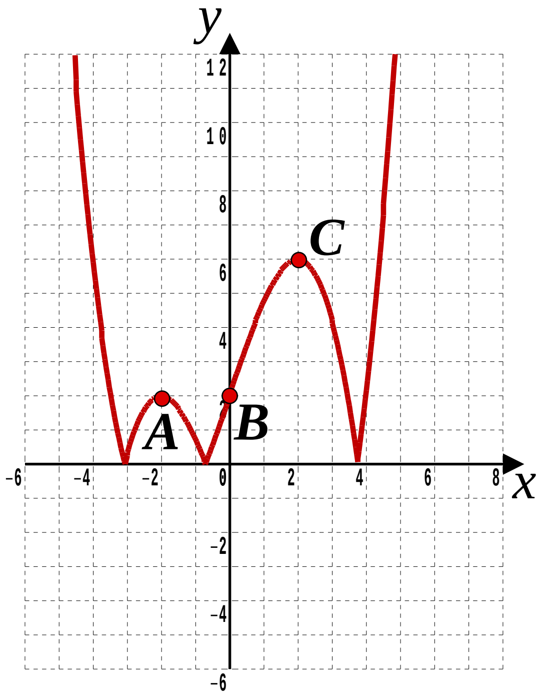
$$pq(x) = 0$$
$$x^2 - 6x - 16 = 0$$
$$(x - 8)(x + 2) = 0$$

Either $x = 8$ or $x = -2$

[2 marks]

Answer 6

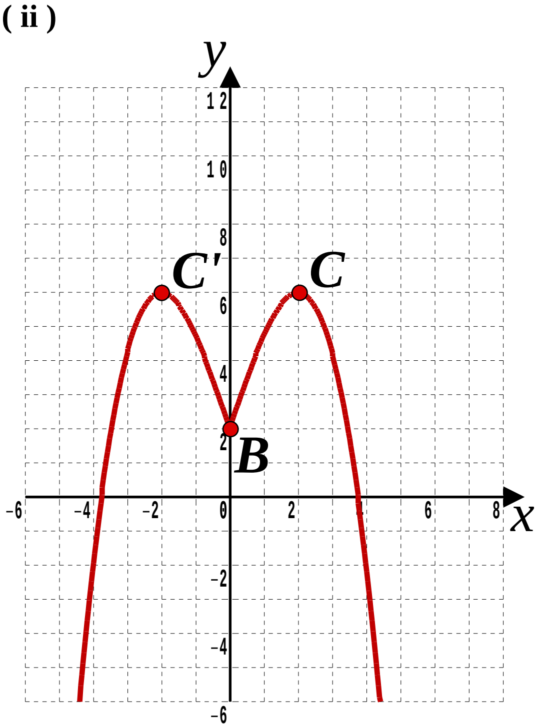
(i)



$A(-2,\ 2)$, $B(0,\ 2)$, $C(2,\ 6)$

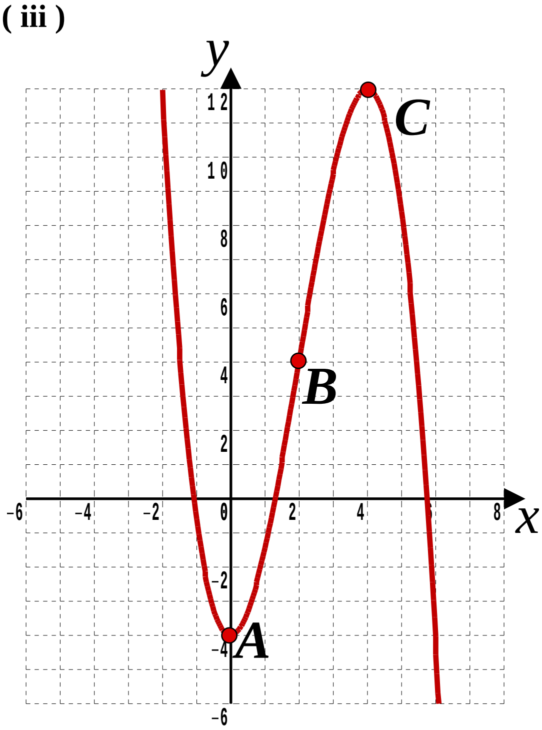
[3 marks]

(ii)



A is lost, $B(0,\ 2)$, $C(2,\ 6)$, $C'(-2,\ 6)$

(iii)



$A(0,\ -4)$, $B(2,\ 4)$, $C(4,\ 12)$

[3, 4 marks]

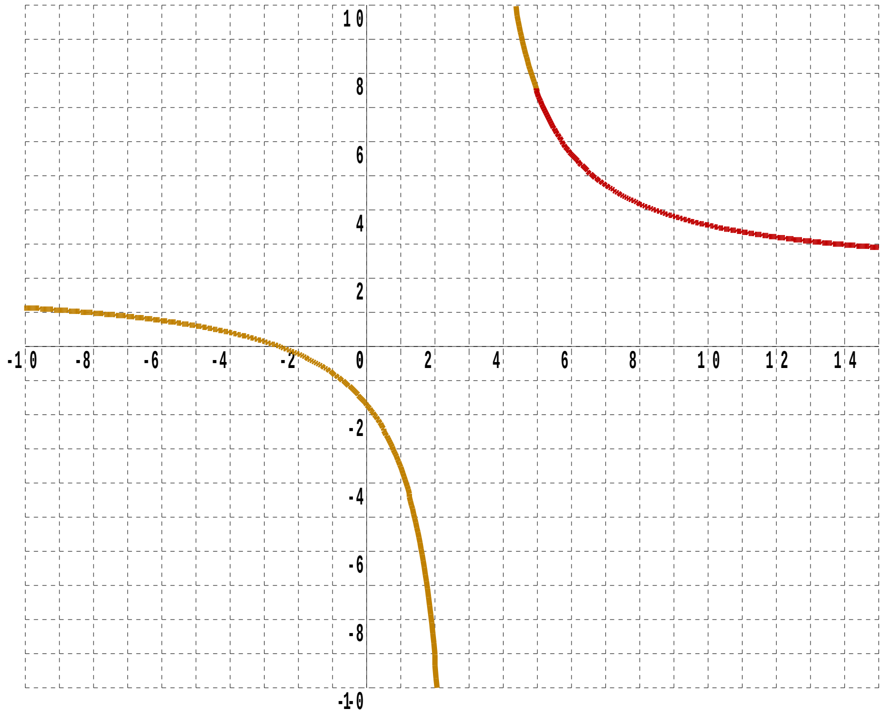
Answer 7

(a)

$$\begin{aligned} gg(5) &= g\left(\frac{2 \times 5 + 5}{5 - 3}\right) \\ &= g\left(\frac{15}{2}\right) \\ &= g(7.5) \\ &= \frac{2 \times 7.5 + 5}{7.5 - 3} \\ &= \frac{20}{4.5} \\ &= \frac{40}{9} \end{aligned}$$

[2 marks]

(b)



The red line is the part of the function for which $x \geqslant 5$

There is a horizontal asymptote of $y = 2$

So the range is $2 < g(x) \leqslant 7.5$

[1 mark]

(c)

$$g(x) = \frac{2x + 5}{x - 3} \qquad x \geqslant 5$$

$$y = \frac{2x + 5}{x - 3}$$

Swapping $x \Leftrightarrow y$ because input $\Leftrightarrow$ output

$$x = \frac{2y + 5}{y - 3}$$

$$x(y - 3) = 2y + 5$$

$$xy - 3x = 2y + 5$$

$$xy - 2y = 3x + 5$$

$$y(x - 2) = 3x + 5$$

$$y = \frac{3x + 5}{x - 2}$$

$$g^{-1}(x) = \frac{3x + 5}{x - 2}$$

The domain of the inverse function is the range of the original function

So the domain is $2 < x \leqslant 7.5$

[3 marks]

Answer 8

$$f^{-1}(x) = \frac{2x - 5}{x}$$

$$y = \frac{2x - 5}{x}$$

$$x \Leftrightarrow y$$

$$x = \frac{2y - 5}{y}$$

$$xy = 2y - 5$$

$$2y - xy = 5$$

$$y(2 - x) = 5$$

$$y = \frac{5}{2 - x}$$

$$f(x) = \frac{5}{2 - x} \qquad x \neq 2$$

(a)

$$fg(x) = f\left(\sqrt{2x - k}\right)$$

$$= \frac{5}{2 - \sqrt{2x - k}} \qquad x \geqslant \frac{k}{2},\ x \neq 2 + \frac{k}{2}$$

[3 marks]

(b)

$$gg(10) = 2$$

$$g\left(\sqrt{20 - k}\right) = 2$$

$$\sqrt{2\sqrt{20 - k} - k} = 2$$

$$2\sqrt{20 - k} - k = 4$$

$$2\sqrt{20 - k} = 4 + k$$

$$4(20 - k) = 16 + 8k + k^2$$

$$k^2 + 12k - 64 = 0$$

$$(k + 16)(k - 4) = 0$$

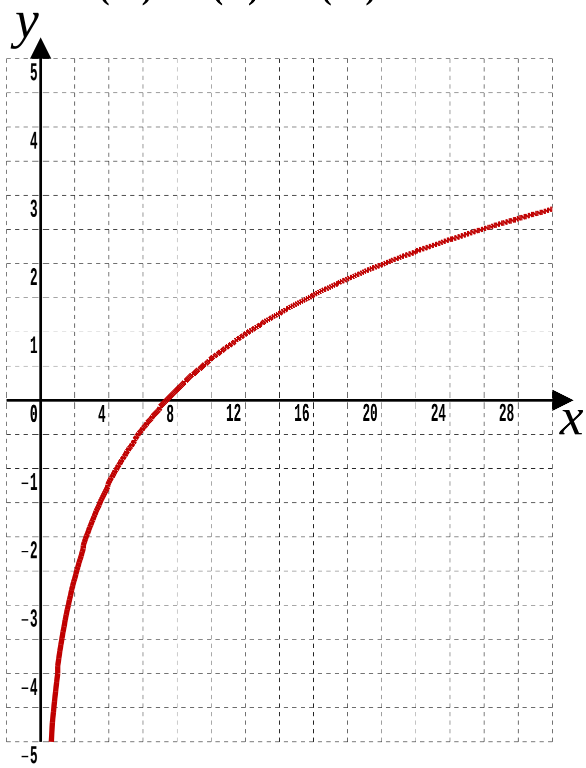
Either $k = -16$ (Reject as told k is positive)

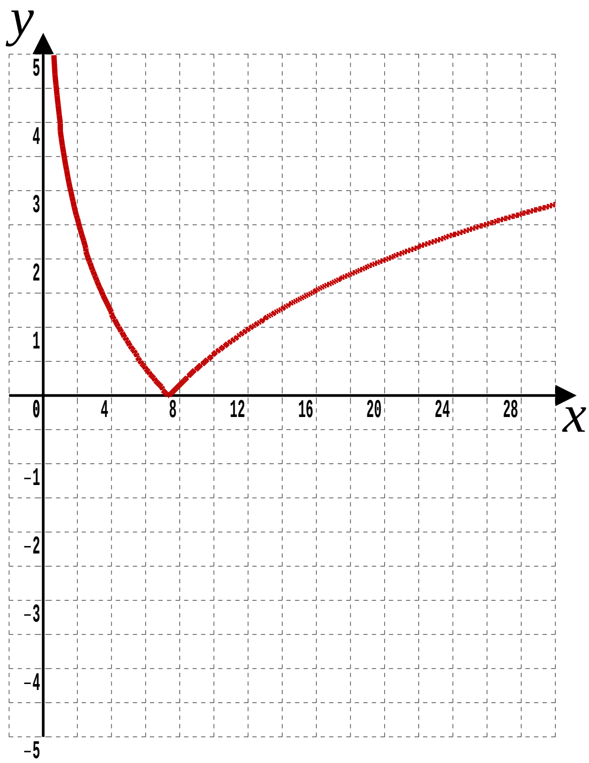
Or $k = 4$

[3 marks]

Answer 9

(a) **(i)** and **(ii)**





x-axis contact, $2\,ln(x) - 4 = 0 \Rightarrow ln(x) = 2 \Rightarrow x = e^2$ (About 7.39)

Asymptote on both graphs is the y-axis which has equation $x = 0$

[5 marks]

(b) $|\,f(x)\,| = 4$

$2\,ln(x) - 4 = 4$ $\quad$ $-\,2\,ln(x) + 4 = 4$

$ln(x) = 4$ $\quad$ $ln(x) = 0$

$x = e^4$ (About 54.6) $\quad$ $x = 1$

[4 marks]

(c)

$$gf(x) = g\big(2\,ln(x) - 4\big)$$
$$= e^{2\,ln\,x - 4 + 5} - 2$$
$$= e^{ln\,x^2 + 1} - 2$$
$$= e\,x^2 - 2$$

[3 marks]

(d) $gf(x) > -2$

[1 mark]