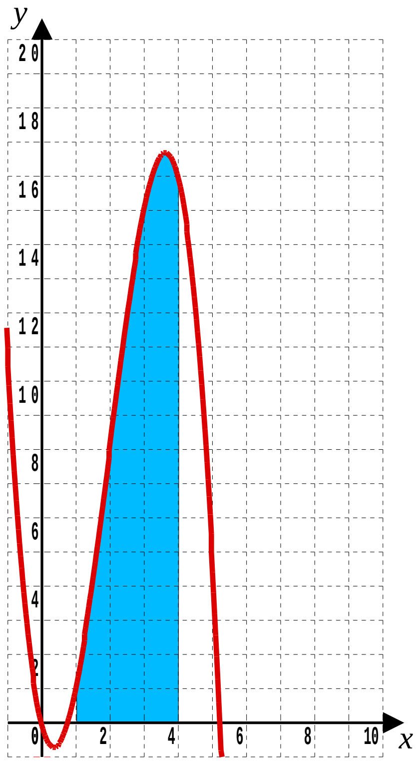
1.7 Solutions

1.7.1 Answer (1.3 Integration Example)



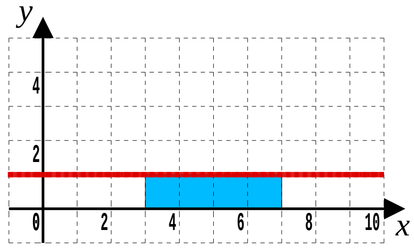
Remainder of solution already provided.

1.7.2 Answer (1.5 Integrating an Isolated Number)

$$\int_3^7 1 \; dx = [\, x \,]_3^7$$

$$= [\,7\,] - [\,3\,]$$

$$= 4$$



Graph is a rectangle of height 1 and width 4 which obviously has an area of 4.

1.7.3 Answers (1.6 Exercise)

Answer 1

$$\int_{2}^{5} 6x^2 + 1 \; dx = \left[\frac{6x^3}{3} + x \right]_2^5$$

$$= \left[2x^3 + x \right]_2^5$$

$$= [2 \times 125 + 5] - [2 \times 8 + 2]$$

$$= [255] - [18]$$

$$= 237 \qquad \square$$

[3 marks]

Answer 2

$$\int_{1}^{3} 24x^3 - 6x \; dx = \left[\frac{24x^4}{4} - \frac{6x^2}{2} \right]_1^3$$

$$= \left[6x^4 - 3x^2 \right]_1^3$$

$$= [6 \times 81 - 3 \times 9] - [6 \times 1 - 3 \times 1]$$

$$= [459] - [3]$$

$$= 456$$

[3 marks]

Answer 3

$$\int_{3}^{5} 36x^2 \; dx = \left[\frac{36x^3}{3} \right]_3^5$$

$$= \left[12x^3 \right]_3^5$$

$$= [12 \times 125] - [12 \times 27]$$

$$= [1500] - [324]$$

$$= 1176$$

[3 marks]

Answer 4

$$\int_1^2 x^4 - 3 \; dx = \left[\frac{x^5}{5} - 3x \right]_1^2$$

$$= \left[\frac{32}{5} - 3 \times 2 \right] - \left[\frac{1}{5} - 3 \times 1 \right]$$

$$= [\, 6.4 - 6 \,] - [\, 0.2 - 3 \,]$$

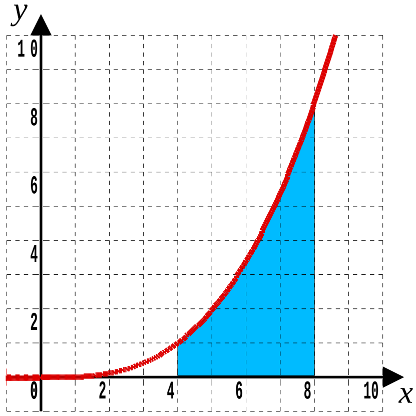
$$= [\, 0.4 \,] - [\, - 2.8 \,]$$

$$= 3.2$$ ☐

[3 marks]

Answer 5

(i)



[1 mark]

(ii)

$$\int_4^8 \frac{x^3}{64} \; dx = \frac{1}{64} \int_4^8 x^3 \, dx$$

$$= \frac{1}{64} \left[\frac{x^4}{4} \right]_4^8$$

$$= \frac{1}{64} \left\{ \left[\frac{8^4}{4} \right] - \left[\frac{4^4}{4} \right] \right\}$$

$$= \frac{1}{64} \{ \, [1024] - [\, 64 \,] \}$$

$$= \frac{1}{64} \{ \, 960 \, \}$$

$$= 15$$ ☐

[3 marks]

Answer 6

$$\int_2^3 (3x+2)(x+1)\ dx$$

$$= \int_2^3 3x^2 + 5x + 2\ dx$$

$$= \left[\frac{3x^3}{3} + \frac{5x^2}{2} + 2x \right]_2^3$$

$$= \left[x^3 + \frac{5x^2}{2} + 2x \right]_2^3$$

$$= \left[27 + \frac{5 \times 9}{2} + 2 \times 3 \right] - \left[8 + \frac{5 \times 4}{2} + 2 \times 2 \right]$$

$$= \left[\frac{111}{2} \right] - \left[\frac{44}{2} \right]$$

$$= 33\frac{1}{2}$$

[4 marks]

Answer 7

$$\int_1^3 x^2 + 3\ dx = \left[\frac{x^3}{3} + 3x \right]_1^3$$

$$= [9 + 9] - \left[\frac{1}{3} + 3 \right]$$

$$= \left[\frac{54}{3} \right] - \left[\frac{10}{3} \right]$$

$$= \frac{44}{3}$$

$$= 14\frac{2}{3}$$

[4 marks]

Answer 8

$$\int_{-1}^4 (1+x)(4-x)\ dx = \int_{-1}^4 4 + 3x - x^2\ dx$$

$$= \left[4x + \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-1}^{4}$$

$$= \left[16 + 24 - \frac{64}{3} \right] - \left[-4 + \frac{3}{2} + \frac{1}{3} \right]$$

$$= \left[\frac{125}{6} \right] = 20\frac{5}{6}$$

[5 marks]

Answer 9

As the curve has mirror symmetry in the *y*-axis, the area to be found is given by

$$2 \times \int_0^2 4 - x^2 \; dx = 2 \times \left[4x - \frac{x^3}{3} \right]_0^2$$

$$= 2 \times \left\{ \left[\frac{24}{3} - \frac{8}{3} \right] - [\, 0 \,] \right\}$$

$$= 2 \times \frac{16}{3}$$

$$= \frac{32}{3}$$

$$= 10\frac{2}{3}$$

[4 marks]

Answer 10

(a) − 1 and 5

[1 mark]

(b)
$$\int_{-1}^{5} (x + 1)(x - 5) \; dx = \int_{-1}^{5} x^2 - 4x - 5 \; dx$$

$$= \left[\frac{x^3}{3} - 2x^2 - 5x \right]_{-1}^{5}$$

$$= \left[\frac{125}{3} - 50 - 25 \right] - \left[-\frac{1}{3} - 2 + 5 \right]$$

$$= -36$$

$$\therefore Area = 36$$

[6 marks]

Answer 11

$$\int_1^2 x^3 - x^2 + 3 \; dx = \left[\frac{x^4}{4} - \frac{x^3}{3} + 3x \right]_1^2$$

$$= \left[\frac{2^4}{4} - \frac{2^3}{3} + 3 \times 2 \right] - \left[\frac{1^4}{4} - \frac{1^3}{3} + 3 \times 1 \right]$$

$$= \left[4 - \frac{8}{3} + 6 \right] - \left[\frac{1}{4} - \frac{1}{3} + 3 \right]$$

$$= \frac{48 - 32 + 72 - 3 + 4 - 36}{12}$$

$$= \frac{124 - 71}{12}$$

$$= \frac{53}{12} = 4\frac{5}{12}$$

[1, 4 marks]

Answer 12

$$\int_1^5 \frac{1}{x^3}\ dx = \int_1^5 x^{-3}\ dx$$

$$= \left[\frac{x^{-2}}{-2} \right]_1^5$$

$$= \left[-\frac{1}{2x^2} \right]_1^5$$

$$= \left[-\frac{1}{50} \right] - \left[-\frac{1}{2} \right]$$

$$= \left[\frac{-1 + 25}{50} \right]$$

$$= \frac{24}{50}$$

$$= \frac{12}{25}$$ ☐

[5 marks]