

2.4 Answers to 2.3 Exercise

A-Level Pure Mathematics, Year 1 Additional Mathematics

Integration I

Answer 1

(i)
$$\begin{aligned}\text{LHS} &= \int_1^2 y \, dx \\&= \int_1^2 x^4 - 5x^2 + 4 \, dx \\&= \left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x \right]_1^2 \\&= \left[\left(\frac{32}{5} - \frac{5 \times 8}{3} + 4 \times 2 \right) - \left(\frac{1}{5} - \frac{5}{3} + 4 \right) \right] \\&= \left(\frac{16}{15} \right) - \left(\frac{38}{15} \right) \\&= -\frac{22}{15} \\&= \text{RHS} \quad \square\end{aligned}$$

[5 marks]

(ii)
$$\begin{aligned}\int_0^1 y \, dx &= \int_0^1 x^4 - 5x^2 + 4 \, dx \\&= \left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x \right]_0^1 \\&= \left(\frac{1}{5} - \frac{5}{3} + 4 \right) - (0) \\&= \frac{38}{15}\end{aligned}$$

[2 marks]

(iii)
$$\begin{aligned}\text{Area shaded} &= \frac{22}{15} + \frac{38}{15} \\&= 4\end{aligned}$$

[1 mark]

Answer 2

$$\begin{aligned}
 \text{(i)} \quad \int_0^4 y \, dx &= \int_0^4 x(x-2)(x-4) \, dx \\
 &= \int_0^4 x(x^2 - 6x + 8) \, dx \\
 &= \int_0^4 x^3 - 6x^2 + 8x \, dx \\
 &= \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{8x^2}{2} \right]_0^4 \\
 &= \left[\frac{x^4}{4} - 2x^3 + 4x^2 \right]_0^4 \\
 &= \left[\frac{256}{4} - 2 \times 64 + 4 \times 16 \right] - [0] \\
 &= 0
 \end{aligned}$$

[4 marks]**(ii)** The two areas must be the same**[1 mark]****Answer 3**

$$\begin{aligned}
 \int_1^4 6x^2 - 5x^4 \, dx &= \left[\frac{6x^3}{3} - \frac{5x^5}{5} \right]_1^4 \\
 &= \left[2x^3 - x^5 \right]_1^4 \\
 &= [2 \times 4^3 - 4^5] - [2 \times 1^3 - 1^5] \\
 &= [128 - 1024] - [1] \\
 &= -897
 \end{aligned}$$

[4 marks]

Answer 4**(i)**

$$\int_1^a (5 - 2x) dx = 0$$

$$\left[5x - x^2 \right]_1^a = 0$$

$$\left[5a - a^2 \right] - \left[5 - 1 \right] = 0$$

$$a^2 - 5a + 4 = 0$$

$$(a - 4)(a - 1) = 0$$

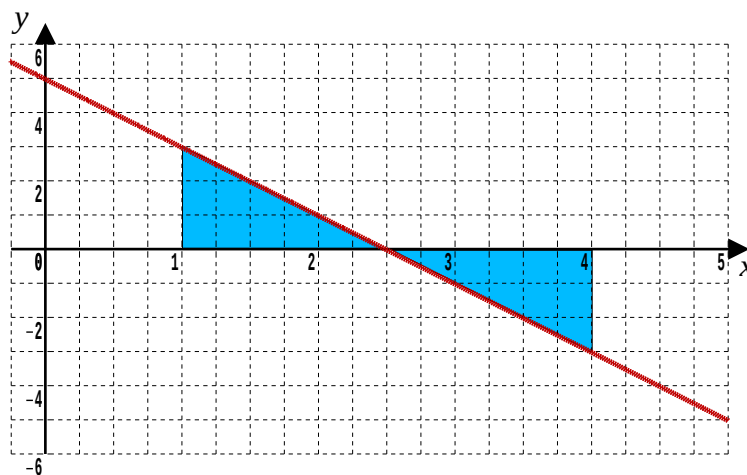
$a = 4$ is the only sensible solution

[4 marks]

(ii) The integration is finding the area under the 'curve' which, in this case, is the straight line $y = 5 - 2x$.

However, areas under the x-axis are seen as negative and so the upper limit is being moved to a place where the area above the x-axis is the same as the area below.

The graph below shows the situation when $a = 4$

**[2 marks]**

Answer 5

$$\begin{aligned}
 \text{(i)} \quad \text{LHS} &= \int_0^2 x^2 + 2x - 3 \, dx \\
 &= \left[\frac{x^3}{3} + \frac{2x^2}{2} - 3x \right]_0^2 \\
 &= \left[\frac{x^3}{3} + x^2 - 3x \right]_0^2 \\
 &= \left[\frac{8}{3} + 4 - 6 \right] - [0] \\
 &= \frac{2}{3} \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

[3 marks]

- (ii)** Marc has forgotten that integration will 'see' area under the x -axis as a negative quantity.
 As the curve crosses the x -axis between $x = 0$ and $x = 1$ this will be an issue with some 'negative area' giving an incorrect result

[1 mark]

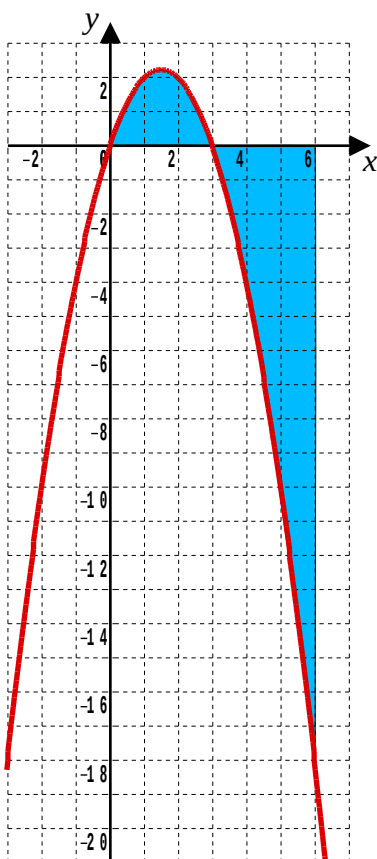
- (iii)** Area between $x = 0$ and $x = 1$;

$$\begin{aligned}
 \int_0^1 x^2 + 2x - 3 \, dx &= \left[\frac{x^3}{3} + x^2 - 3x \right]_0^1 \\
 &= \frac{1}{3} + 1 - 3 \\
 &= -\frac{5}{3}
 \end{aligned}$$

Total area requested will be,

$$\frac{2}{3} + 2 \times \frac{5}{3} = 4$$

[3 marks]

Answer 6**(i)**

As the curve crosses the x -axis between $x = 0$ and $x = 6$ the integration over the whole interval in one go will not correspond to the area under the curve

(ii)

$$\begin{aligned}\int_0^3 3x - x^2 \, dx &= \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3 \\ &= \left[\frac{27}{2} - 9 \right] - [0] \\ &= 4.5\end{aligned}$$

$$\begin{aligned}\int_3^6 3x - x^2 \, dx &= \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_3^6 \\ &= [54 - 72] - [4.5] \\ &= [-18] - [4.5] \\ &= -22.5\end{aligned}$$

$\therefore$ Area specified is 27

[6 marks]

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