2.4 Answers to 2.3 Exercise

Answer 1

(i)

$$\begin{aligned}
\text{LHS} &= \int_1^2 y \ dx \\
&= \int_1^2 x^4 - 5x^2 + 4 \ dx \\
&= \left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x \right]_1^2 \\
&= \left[\left(\frac{32}{5} - \frac{5 \times 8}{3} + 4 \times 2 \right) - \left(\frac{1}{5} - \frac{5}{3} + 4 \right) \right] \\
&= \left(\frac{16}{15} \right) - \left(\frac{38}{15} \right) \\
&= -\frac{22}{15} \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[5 marks]

(ii)

$$\begin{aligned}
\int_0^1 y \ dx &= \int_0^1 x^4 - 5x^2 + 4 \ dx \\
&= \left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x \right]_0^1 \\
&= \left(\frac{1}{5} - \frac{5}{3} + 4 \right) - (0) \\
&= \frac{38}{15}
\end{aligned}$$

[2 marks]

(iii)

$$\begin{aligned}
\text{Area shaded} &= \frac{22}{15} + \frac{38}{15} \\
&= 4
\end{aligned}$$

[1 mark]

Answer 2

(i)

$$\int_0^4 y\ dx = \int_0^4 x(x-2)(x-4)\ dx$$

$$= \int_0^4 x\left(x^2 - 6x + 8\right)\ dx$$

$$= \int_0^4 x^3 - 6x^2 + 8x\ dx$$

$$= \left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{8x^2}{2}\right]_0^4$$

$$= \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_0^4$$

$$= \left[\frac{256}{4} - 2 \times 64 + 4 \times 16\right] - [\,0\,]$$

$$= 0$$

[4 marks]

(ii) The two areas must be the same

[1 mark]

Answer 3

$$\int_1^4 6x^2 - 5x^4\ dx = \left[\frac{6x^3}{3} - \frac{5x^5}{5}\right]_1^4$$

$$= \left[2x^3 - x^5\right]_1^4$$

$$= \left[2 \times 4^3 - 4^5\right] - \left[2 \times 1^3 - 1^5\right]$$

$$= [\,128 - 1024\,] - [\,1\,]$$

$$= -897$$

[4 marks]

Answer 4

(i)

$$\int_1^a (5 - 2x)\, dx = 0$$

$$\Big[5x - x^2 \Big]_1^a = 0$$

$$\Big[5a - a^2 \Big] - \Big[5 - 1 \Big] = 0$$

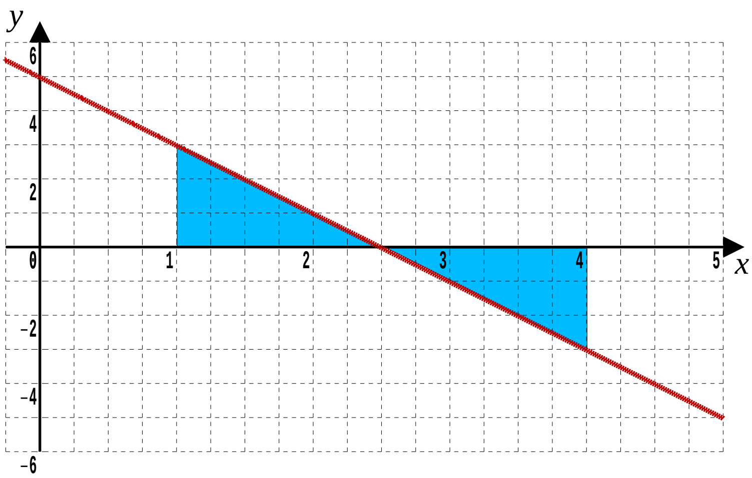
$$a^2 - 5a + 4 = 0$$

$$(a - 4)(a - 1) = 0$$

$a = 4$ is the only sensible solution

[4 marks]

(ii) The integration is finding the area under the 'curve' which, in this case, is the straight line $y = 5 - 2x$.
However, areas under the x-axis are seen as negative and so the upper limit is being moved to a place where the area above the x-axis is the same as the area below.
The graph below shows the situation when $a = 4$



[2 marks]

Answer 5

(i)

$$\text{LHS} = \int_0^2 x^2 + 2x - 3 \; dx$$

$$= \left[\frac{x^3}{3} + \frac{2x^2}{2} - 3x \right]_0^2$$

$$= \left[\frac{x^3}{3} + x^2 - 3x \right]_0^2$$

$$= \left[\frac{8}{3} + 4 - 6 \right] - [\, 0 \,]$$

$$= \frac{2}{3}$$

$$= \text{RHS} \qquad \square$$

[3 marks]

(ii) Marc has forgotten that integration will 'see' area under the x-axis as a negative quantity.
As the curve crosses the x-axis between $x = 0$ and $x = 1$ this will be an issue with some 'negative area' giving an incorrect result

[1 mark]

(iii) Area between $x = 0$ and $x = 1$;

$$\int_0^1 x^2 + 2x - 3 \; dx = \left[\frac{x^3}{3} + x^2 - 3x \right]_0^1$$

$$= \frac{1}{3} + 1 - 3$$

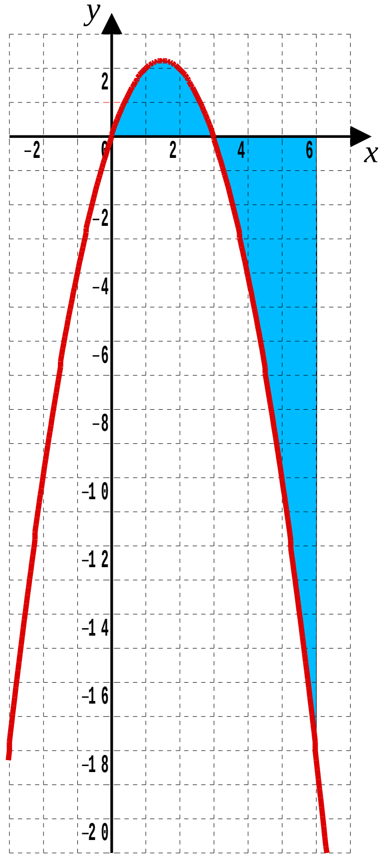
$$= -\frac{5}{3}$$

Total area requested will be,

$$\frac{2}{3} + 2 \times \frac{5}{3} = 4$$

[3 marks]

Answer 6
(i)



As the curve crosses the x-axis between $x = 0$ and $x = 6$ the integration over the whole interval in one go will not correspond to the area under the curve

(ii)

$$\int_0^3 3x - x^2 \; dx = \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3$$

$$= \left[\frac{27}{2} - 9 \right] - [\,0\,]$$

$$= 4.5$$

$$\int_3^6 3x - x^2 \; dx = \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_3^6$$

$$= [\,54 - 72\,] - [\,4.5\,]$$

$$= [\,-18\,] - [\,4.5\,]$$

$$= -22.5$$

$\therefore$ Area specified is 27

[6 marks]