3.4 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
Integration I

3.4.1 Solution to 3.2 "Fractional Powers" Example (Teaching Video)

$$\int_1^{16} x^{\frac{3}{4}}\,dx = \left[\frac{4x^{\frac{7}{4}}}{7}\right]_1^{16}$$

$$= \frac{4}{7}\left[x^{\frac{7}{4}}\right]_1^{16}$$

$$= \frac{4}{7}\left[16^{\frac{7}{4}} - 1^{\frac{7}{4}}\right]$$

$$= \frac{4}{7}\left[128 - 1\right]$$

$$= \frac{508}{7} \qquad = 72\frac{4}{7}$$

3.4.2 Answers to 3.3 Exercise

Answer 1

$$\int_4^{25} x^{\frac{1}{2}}\,dx = \left[\frac{2x^{\frac{3}{2}}}{3}\right]_4^{25}$$

$$= \frac{2}{3}\left[x^{\frac{3}{2}}\right]_4^{25}$$

$$= \frac{2}{3}\left[25^{\frac{3}{2}} - 4^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[125 - 8\right]$$

$$= 78$$

[3 marks plus 1 bonus mark]

Answer 2

$$\int_0^3 \sqrt{3x}\, dx = \sqrt{3} \int_0^3 x^{\frac{1}{2}}\, dx$$

$$= \sqrt{3}\left[\frac{2x^{\frac{3}{2}}}{3} \right]_0^3$$

$$= \frac{2\sqrt{3}}{3}\left[x^{\frac{3}{2}} \right]_0^3$$

$$= \frac{2\sqrt{3}}{3}\left[3^{\frac{3}{2}} - 0 \right]$$

$$= \frac{2\sqrt{3}}{3} \times 3\sqrt{3}$$

$$= 6$$

[4 marks]

Answer 3

$$\int_4^{25} \frac{5}{3\sqrt{x}}\, dx = \frac{5}{3} \int_4^{25} x^{-\frac{1}{2}}\, dx$$

$$= \frac{5}{3}\left[\frac{2x^{\frac{1}{2}}}{1} \right]_4^{25}$$

$$= \frac{10}{3}\left[\sqrt{x} \right]_4^{25}$$

$$= \frac{10}{3}\left\{ [5] - [2] \right\}$$

$$= \frac{10}{3}\left\{ 3 \right\}$$

$$= 10$$

[5 marks]

Answer 4

$$\int_1^4 y\ dx = \int_1^4 \frac{\sqrt{x} + 1}{3}\ dx$$

$$= \frac{1}{3} \int_1^4 \sqrt{x} + 1\ dx$$

$$= \frac{1}{3} \int_1^4 x^{\frac{1}{2}} + 1\ dx$$

$$= \frac{1}{3} \left[\frac{2x^{\frac{3}{2}}}{3} + x \right]_1^4$$

$$= \frac{1}{3}\left[\frac{2 \times 4^{\frac{3}{2}}}{3} + 4 - \left(\frac{2 \times 1^{\frac{3}{2}}}{3} + 1 \right) \right]$$

$$= \frac{1}{3}\left[\frac{16}{3} + 4 - \frac{2}{3} - 1 \right]$$

$$= \frac{1}{3}\left[\frac{23}{3} \right]$$

$$= \frac{23}{9} \qquad \text{or} \qquad 2\frac{5}{9}$$

[5 marks]

Answer 5

$$\int_1^4 \left(2x + 3\sqrt{x} \right) dx = \int_1^4 \left(2x + 3x^{\frac{1}{2}} \right) dx$$

$$= \left[x^2 + 2x^{\frac{3}{2}} \right]_1^4$$

$$= \{ [16 + 16] - [1 + 2] \}$$

$$= 29$$

[5 marks]

Answer 6

$$\int_2^4 \left(\frac{4}{\sqrt{x}} + k \right) dx = 30$$

$$\int_2^4 4x^{-\frac{1}{2}} + k\ dx = 30$$

$$\left[8x^{\frac{1}{2}} + kx \right]_2^4 = 30$$

$$16 + 4k - 8\sqrt{2} - 2k = 30$$

$$2k = 30 - 16 + 8\sqrt{2}$$

$$k = 7 + 4\sqrt{2}$$

[6 marks]

Answer 7

$$\int_1^9 x^{\frac{1}{2}} - 3 \; dx = \left[\frac{2x^{\frac{3}{2}}}{3} - 3x \right]_1^9$$

$$= \left\{ \left[\frac{2 \times 27}{3} - 3 \times 9 \right] - \left[\frac{2 \times 1}{3} - 3 \times 1 \right] \right\}$$

$$= \left\{ [\, 18 - 27 \,] - \left[\frac{2}{3} - 3 \right] \right\}$$

$$= \left\{ [-9\,] - \left[-\frac{7}{3} \right] \right\}$$

$$= \left[-9 + \frac{7}{3} \right]$$

$$= -\frac{20}{3} \qquad \therefore \; Area = \frac{20}{3}$$

[6 marks]

Answer 8

$$\int_1^8 \frac{1}{\sqrt{x}} \; dx = \int_1^8 x^{-\frac{1}{2}} \; dx$$

$$= \left[\frac{2x^{\frac{1}{2}}}{1} \right]_1^8$$

$$= \left[2\sqrt{x} \right]_1^8$$

$$= \left\{ [2\sqrt{8}\,] - [\,2\,] \right\}$$

$$= \left\{ [2 \times 2\sqrt{2}\,] - [\,2\,] \right\}$$

$$= -2 + 4\sqrt{2}$$

[4 marks]