4.3 Answers

4.3.1 Solution to 4.1 Example (Teaching Video)

The two curves have the equations,

$y = 8x - x^2$ The red 'upside down' parabola

$y = x^2$ The gold parabola

Find area between the two curves.

To find the x values of where the two curves intersect,

$$x^2 = 8x - x^2$$

$$2x^2 - 8x = 0$$

Divide through by 2

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

Either $x = 0$ or $x = 4$

$$Area = \int_0^4 8x - x^2\,dx - \int_0^4 x^2\,dx$$

$$= \int_0^4 8x - x^2 - x^2\,dx$$

$$= \int_0^4 8x - 2x^2\,dx$$

$$= \left[4x^2 - \frac{2x^3}{3} \right]_0^4$$

$$= 4 \times 4^2 - \frac{2 \times 4^3}{3}$$

$$= 64 - \frac{128}{3}$$

$$= \frac{64}{3}$$

$$= 21\frac{1}{3}$$

[6 marks]

4.3.2 Solutions to 4.2 Exercise

Answer 1

(i)

$$x^2 - 4x + 6 = 4x - x^2$$

$$2x^2 - 8x + 6 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x - 1)(x - 3) = 0$$

$$\therefore \text{Either } x = 1 \text{ or } x = 3$$

[3 marks]

(ii)

$$Area = \int_1^3 4x - x^2\,dx - \int_1^3 x^2 - 4x + 6\,dx$$

$$= \int_1^3 4x - x^2 - x^2 + 4x - 6\,dx$$

$$= \int_1^3 8x - 2x^2 - 6\,dx$$

[2 marks]

(iii)

$$\int_1^3 8x - 2x^2 - 6\,dx$$

$$= \left[4x^2 - \frac{2x^3}{3} - 6x \right]_1^3$$

$$= \left\{ \left[4 \times 9 - \frac{2 \times 27}{3} - 6 \times 3 \right] - \left[4 \times 1 - \frac{2}{3} - 6 \times 1 \right] \right\}$$

$$= (36 - 18 - 18) - (4 - \frac{2}{3} - 6)$$

$$= -\left(-\frac{8}{3}\right)$$

$$= 2\frac{2}{3}$$

[4 marks]

Answer 2

(a)

$$x^2 + 6x + 10 = 3x + 20$$

$$x^2 + 3x - 10 = 0$$

$$(x + 5)(x - 2) = 0$$

$$\therefore A(-5,\ 5) \text{ and } B(2,\ 26)$$

[5 marks]

(b)

$$Area = \int_{-5}^{2} 3x + 20 - \int_{-5}^{2} x^2 + 6x + 10\ dx$$

$$= \int_{-5}^{2} 3x + 20 - x^2 - 6x - 10\ dx$$

$$= \int_{-5}^{2} 10 - 3x - x^2\ dx$$

$$= \left[10x - \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-5}^{2}$$

$$= \left\{ \left[20 - 6 - \frac{8}{3} \right] - \left[-50 - \frac{75}{2} + \frac{125}{3} \right] \right\}$$

$$= \left\{ \left[\frac{34}{3} \right] - \left[-\frac{275}{6} \right] \right\}$$

$$= \frac{343}{6} \quad \text{or} \quad 57\frac{1}{6}$$

[7 marks]

Answer 3

(i)

$$x^2 - 6x + 11 = -x^2 + 8x - 9$$

$$2x^2 - 14x + 20 = 0$$

$$x^2 - 7x + 10 = 0$$

$$(x - 2)(x - 5) = 0$$

$$\therefore A(2, 3) \text{ and } B(5, 6)$$

[5 marks]

(ii)

$$Area = \int_2^5 -x^2 + 8x - 9 \, dx - \int_2^5 x^2 - 6x + 11 \, dx$$

$$= \int_2^5 -2x^2 + 14x - 20 \, dx$$

$$= \left[-\frac{2x^3}{3} + 7x^2 - 20x \right]_2^5$$

$$= \left\{ \left[-\frac{250}{3} + 175 - 100 \right] - \left[-\frac{16}{3} + 28 - 40 \right] \right\}$$

$$= \left[-\frac{25}{3} \right] - \left[-\frac{52}{3} \right]$$

$$= 9$$

[7 marks]

Answer 4

(i) $x^2 - 4x + 7 = 6x - x^2 - 1$

$2x^2 - 10x + 8 = 0$

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

Either $x = 1$ or $x = 4$

When $x = 1$: $x^2 - 4x + 7 = 1^2 - 4 \times 1 + 7$

$= 4 \quad \Rightarrow A(1, 4)$ □

When $x = 4$: $x^2 - 4x + 7 = 4^2 - 4 \times 4 + 7$

$= 7 \quad \Rightarrow B(4, 7)$ □

[2 marks]

(ii) $PQ = \left(6x - x^2 - 1\right) - \left(x^2 - 4x + 7\right)$

$= -2x^2 + 10x - 8 \quad$ for $1 \leqslant x \leqslant 4$

[2 marks]

(iii) If $y = -2x^2 + 10x - 8$ then $\frac{dy}{dx} = -4x + 10$

For the greatest length, $\frac{dy}{dx} = 0 \quad \therefore \; -4x + 10 = 0$

$x = \frac{5}{2}$

Thus, greatest length of $PQ = -2 \times \left(\frac{5}{2}\right)^2 + 10 \times \left(\frac{5}{2}\right) - 8$

$= -\frac{25}{2} + 25 - 8$

$= \frac{9}{2}$

[4 marks]

(iv) $Area = \int_1^4 -2x^2 + 10x - 8 \; dx$

$= \left[-\frac{2x^3}{3} + 5x^2 - 8x\right]_1^4$

$= \left[-\frac{128}{3} + 80 - 32\right] - \left[-\frac{2}{3} + 5 - 8\right]$

$= \left[\frac{16}{3}\right] - \left[-\frac{11}{3}\right]$

$= 9$

[4 marks]

Answer 5

(i)

$$y = 16x + \frac{27}{x^2}$$

$$= 16x + 27x^{-2}$$

$$\frac{dy}{dx} = 16 - 54x^{-3}$$

$$= 16 - \frac{54}{x^3}$$

$$\frac{dy}{dx} = 0 \text{ when } 16 - \frac{54}{x^3} = 0$$

$$16 = \frac{54}{x^3}$$

$$16x^3 = 54$$

$$x^3 = \frac{27}{8}$$

$$x = \frac{3}{2}$$

$$\text{When } x = \frac{3}{2},\ y = 16 \times \left(\frac{3}{2}\right) + \frac{27}{\left(\frac{3}{2}\right)^2}$$

$$= 24 + 12$$

$$= 36 \quad \Rightarrow \quad A\left(\frac{3}{2},\ 36\right)$$

[4 marks]

(ii)

$$P(1,\ 43) \text{ and } Q(3,\ 51)$$

$$Area = \frac{1}{2}(43 + 51) \times 2 - \int_1^3 16x + 27x^{-2}\,dx$$

$$= 94 - \left[8x^2 + \frac{27x^{-1}}{(-1)}\right]_1^3$$

$$= 94 - \left[8x^2 - \frac{27}{x}\right]_1^3$$

$$= 94 - [72 - 9] + [8 - 27]$$

$$= 12$$

[6 marks]

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