

5.5 Answers to 5.4 Exercise

A-Level Pure Mathematics, Year 1 Additional Mathematics Integration I

Answer 1

$$(i) \quad \int x^4 dx = \frac{x^5}{5} + c$$

$$(ii) \quad \int 8x^3 dx = \frac{8x^4}{4} + c \\ = 2x^4 + c$$

$$(iii) \quad \int 6x^{-3} dx = \frac{6x^{-2}}{(-2)} + c \\ = -\frac{3}{x^2} + c$$

$$(iv) \quad \int \frac{5}{x^2} dx = \int 5x^{-2} dx \\ = \frac{5x^{-1}}{(-1)} + c \\ = -\frac{5}{x} + c$$

$$(v) \quad \int x^{\frac{3}{5}} dx = \frac{5x^{\frac{8}{5}}}{8} + c$$

$$(vi) \quad \int 12\sqrt{x} dx = \int 12x^{\frac{1}{2}} dx \\ = \frac{12 \times 2x^{\frac{3}{2}}}{3} + c \\ = 8x^{\frac{3}{2}} + c$$

$$(vii) \quad \int x + 5 dx = \frac{x^2}{2} + 5x + c$$

$$(viii) \quad \int \frac{x^3 + x^2}{x} dx = \int \frac{x^3}{x} + \frac{x^2}{x} dx \\ = \int x^2 + x dx \\ = \frac{x^3}{3} + \frac{x^2}{2} + c$$

[16 marks]

Answer 2

$$\int \left(6x^2 + \frac{2}{x^2} + 5 \right) dx = \int (6x^2 + 2x^{-2} + 5) dx \\ = \frac{6x^3}{3} + \frac{2x^{-1}}{(-1)} + 5x + c \\ = 2x^3 - \frac{2}{x} + 5x + c$$

[4 marks]

Answer 3**(a)**

$$y = x^4 + 6x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 4x^3 + 3x^{-\frac{1}{2}}$$

$$= 4x^3 + \frac{3}{\sqrt{x}}$$

[3 marks]**(b)**

$$y = x^4 + 6x^{\frac{1}{2}}$$

$$\int y \, dx = \int x^4 + 6x^{\frac{1}{2}} \, dx$$

$$= \frac{x^5}{5} + \frac{6 \times 2x^{\frac{3}{2}}}{3} + c$$

$$= \frac{x^5}{5} + 4x^{\frac{3}{2}} + c$$

[3 marks]**Answer 4****(a)**

$$y = 2x^5 + 7 + \frac{1}{x^3}$$

$$y = 2x^5 + 7 + x^{-3}$$

$$\frac{dy}{dx} = 10x^4 - 3x^{-4}$$

$$\frac{dy}{dx} = 10x^4 - \frac{3}{x^4}$$

[3 marks]**(b)**

$$y = 2x^5 + 7 + \frac{1}{x^3}$$

$$\int y \, dx = \int 2x^5 + 7 + x^{-3} \, dx$$

$$= \frac{2x^6}{6} + 7x + \frac{x^{-2}}{(-2)} + c$$

$$= \frac{x^6}{3} + 7x - \frac{1}{2x^2} + c$$

[4 marks]

Answer 5

$$\begin{aligned}\int \left(12x^5 - 3x^2 + 4x^{\frac{1}{3}} \right) dx &= \frac{12x^6}{6} - \frac{3x^3}{3} + \frac{4 \times 3x^{\frac{4}{3}}}{4} + c \\ &= 2x^6 - x^3 + 3x^{\frac{4}{3}} + c\end{aligned}$$

[5 marks]

Answer 6

(i)

$$\begin{aligned}y &= 3x - x^2 \\ \frac{dy}{dx} &= 3 - 2x\end{aligned}$$

[2 marks]

(ii)

Gradient at B , where $x = 1$ is thus 1

$$\begin{aligned}y &= mx + c \\ 2 &= 1 \times 1 + c \Rightarrow c = 1 \\ \therefore y &= x + 1\end{aligned}$$

[3 marks]

(iii)

$$A(-1, 0)$$

[1 mark]

(iv)

$$\begin{aligned}\text{Area sail} &= \text{area } \Delta - \text{area under curve} \\ &= \frac{1}{2} \times 2 \times 2 - \int_0^1 3x - x^2 dx \\ &= 2 - \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^1 \\ &= 2 - \left\{ \left[\frac{3}{2} - \frac{1}{3} \right] - [0] \right\} \\ &= 2 - \frac{7}{6} \\ &= \frac{5}{6} \text{ m}^2\end{aligned}$$

[6 marks]

Answer 7**(a)**

$$x^2 - 5x + 4 = 0$$

$$(x - 1)(x - 4) = 0$$

Either $x = 1$ or $x = 4$

Thus, $L(1, 0)$ and $M(4, 0)$

[2 marks]**(b)**

$$y = x^2 - 5x + 4$$

$$= 5^2 - 5 \times 5 + 4$$

$$= 4 \quad \text{so } N(5, 4) \text{ lies on } C$$

[1 mark]**(c)**

$$\int (x^2 - 5x + 4) dx = \frac{x^3}{3} - \frac{5x^2}{2} + 4x + c$$

[2 marks]**(d)**

$$\text{Area} = \frac{1}{2} \times 4 \times 4 - \int_4^5 x^2 - 5x + 4 dx$$

$$= 8 - \left[\frac{x^3}{3} - \frac{5x^2}{2} + 4x \right]_4^5$$

$$= 8 - \left\{ \left[\frac{5^3}{3} - \frac{5 \times 5^2}{2} + 4 \times 5 \right] - \left[\frac{4^3}{3} - \frac{5 \times 4^2}{2} + 4 \times 4 \right] \right\}$$

$$= 8 - \left\{ \left[\frac{2 \times 125}{6} - \frac{3 \times 125}{6} + 20 \right] - \left[\frac{64}{3} - 40 + 16 \right] \right\}$$

$$= 8 - \left\{ \left[-\frac{125}{6} + 20 \right] - \left[21\frac{1}{3} - 24 \right] \right\}$$

$$= 8 - \left\{ \left[-20\frac{5}{6} + 20 \right] - \left[-2\frac{2}{3} \right] \right\}$$

$$= 8 - \left\{ -\frac{5}{6} + \frac{16}{6} \right\}$$

$$= 8 - \frac{11}{6}$$

$$= 6\frac{1}{6}$$

[5 marks]

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