6.4 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
Integration I

6.4.1 Solution to 6.2 Example (Teaching Video)

$$y = \int 12\,x^5 + 3\,dx$$

$$y = \frac{12\,x^6}{6} + 3x + c$$

$$y = 2\,x^6 + 3x + c$$

Told that this curve passes through (1, 9)

$$9 = 2 \times 1^6 + 3 \times 1 + c$$

$$9 = 5 + c$$

$$c = 4$$

Thus, the equation of the curve is

$$y = 2\,x^6 + 3x + 4$$

6.4.2 Solutions to 6.3 Exercise

Answer 1

$$y = \int \frac{dy}{dx}\,dx$$

$$= \int 3\,x^2 - 2x + 4\,dx$$

$$= \frac{3\,x^3}{3} - \frac{2\,x^2}{2} + 4x + c$$

$$= x^3 - x^2 + 4x + c$$

Told that this curve passes through (2, 2)

$$2 = 2^3 - 2^2 + 4 \times 2 + c$$

$$2 = 8 - 4 + 8 + c$$

$$c = -10$$

Thus, the equation of the curve is

$$y = x^3 - x^2 + 4x - 10$$

[4 marks]

Answer 2

$$y = \int \frac{dy}{dx}\, dx$$

$$= \int 2 + 2x - 3x^2 \; dx$$

$$= 2x + \frac{2x^2}{2} - \frac{3x^3}{3} + c$$

$$= 2x + x^2 - x^3 + c$$

Told that this curve passes through (2, 3)

$$3 = 2 \times 2 + 2^2 - 2^3 + c$$

$$3 = 4 + 4 - 8 + c$$

$$c = 3$$

Thus, the equation of the curve is

$$y = 2x + x^2 - x^3 + 3$$

[4 marks]

Answer 3

$$\int f'(x) \; dx = \int 3x^2 - 3x + 5\, dx$$

$$f(x) = \frac{3x^3}{3} - \frac{3x^2}{2} + 5x + c$$

$$y = x^3 - \frac{3x^2}{2} + 5x + c$$

Told that this curve passes through (2, 10)

$$10 = 2^3 - \frac{3 \times 2^2}{2} + 5 \times 2 + c$$

$$10 = 8 - 6 + 10 + c$$

$$c = -2$$

Thus, the equation of the curve is

$$y = x^3 - \frac{3x^2}{2} + 5x - 2$$

Finally;

$$f(1) = 1^3 - \frac{3 \times 1^2}{2} + 5 \times 1 - 2$$

$$= 2.5$$

[5 marks]

Answer 4

$$y = \frac{2x^{\frac{2}{3}} + 3}{6}$$

$$= \frac{2x^{\frac{2}{3}}}{6} + \frac{3}{6}$$

$$= \frac{1}{3}x^{\frac{2}{3}} + \frac{1}{2}$$

(a)

$$\frac{dy}{dx} = \frac{1}{3} \times \frac{2}{3} x^{-\frac{1}{3}}$$

$$= \frac{2}{9x^{\frac{1}{3}}}$$

[2 marks]

(b)

$$\int y \; dx = \int \frac{x^{\frac{2}{3}}}{3} + \frac{1}{2} \; dx$$

$$= \frac{3x^{\frac{5}{3}}}{3 \times 5} + \frac{x}{2} + c$$

$$= \frac{1}{5}x^{\frac{5}{3}} + \frac{1}{2}x + c$$

[3 marks]

Answer 5

$$\int \frac{2 + 4x^3}{x^2} \; dx = \int \frac{2}{x^2} + \frac{4x^3}{x^2} \; dx$$

$$= \int 2x^{-2} + 4x \; dx$$

$$= 2 \times \frac{x^{-1}}{(-1)} + 4 \times \frac{x^2}{2} + c$$

$$= -\frac{2}{x} + 2x^2 + c$$

[4 marks]

Answer 6

$$y = \int \frac{dy}{dx}\, dx$$

$$= \int -x^3 + \frac{4x - 5}{2x^3}\, dx$$

$$= \int -x^3 + \frac{4x}{2x^3} - \frac{5}{2x^3}\, dx$$

$$= \int -x^3 + 2x^{-2} - \frac{5}{2} x^{-3}\, dx$$

$$= -\frac{x^4}{4} + \frac{2x^{-1}}{(-1)} - \frac{5}{2} \times \frac{x^{-2}}{(-2)} + c$$

$$= -\frac{x^4}{4} - \frac{2}{x} + \frac{5}{4x^2} + c$$

Using the fact that $y = 7$ when $x = 1$ gives,

$$7 = -\frac{1^4}{4} - \frac{2}{1} + \frac{5}{4 \times 1^2} + c$$

$$7 = -\frac{1}{4} - 2 + \frac{5}{4} + c$$

$$7 = 1 - 2 + c$$

$$c = 8$$

$$\therefore\ y = -\frac{x^4}{4} - \frac{2}{x} + \frac{5}{4x^2} + 8$$

[6 marks]

Answer 7

(a)

$$f(x) = \int f'(x)\, dx$$

$$= \int (x - 2)(3x + 4)\, dx$$

$$= \int 3x^2 - 2x - 8\, dx$$

$$= x^3 - x^2 - 8x + c$$

Making use of the point (3, 6) to get that,

$$6 = 3^3 - 3^2 - 8 \times 3 + c$$

$$6 = 27 - 9 - 24 + c$$

$$c = 12$$

$$\therefore\ f(x) = x^3 - x^2 - 8x + 12$$

[5 marks]

(b) Observe that, $f(2) = 2^3 - 2^2 - 8 \times 2 + 12$

$$= 8 - 4 - 16 + 12$$

$$= 0$$

$\therefore\ (x - 2)$ is a factor of $f(x)$

$$
\begin{array}{r|llll}
 & x^2 & + x & - 6 & \\
\hline
x - 2 & x^3 - x^2 & - 8x & + 12 & \\
 & x^3 - 2x^2 & & & \\
\hline
 & x^2 & - 8x & & \\
 & x^2 & - 2x & & \\
\hline
 & & - 6x & + 12 & \\
 & & - 6x & + 12 & \\
\hline
 & & & \color{red}{0} & \leftarrow \text{Remainder, as expected}
\end{array}
$$

$$\therefore\ f(x) = (x - 2)\left(x^2 + x - 6\right)$$

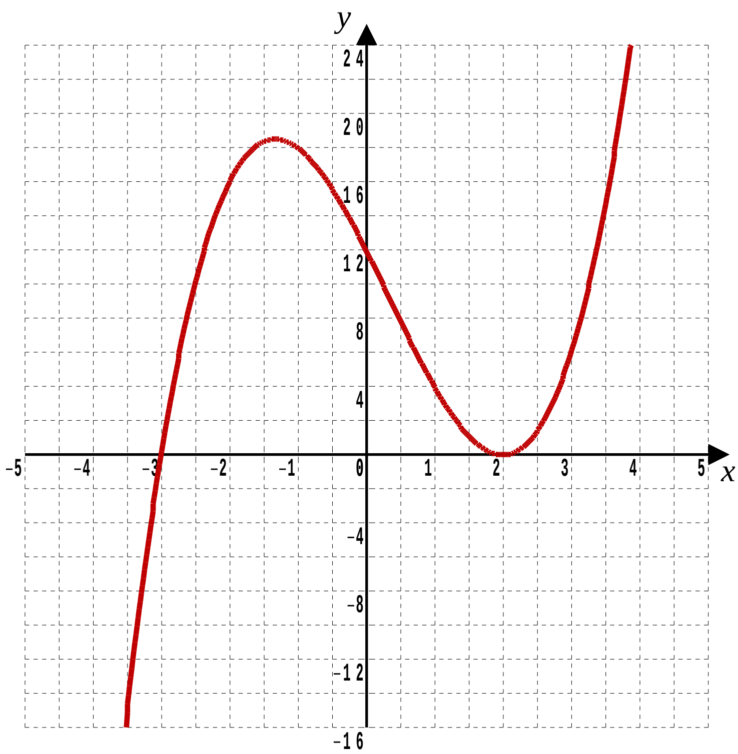
$$= (x - 2)(x + 3)(x - 2)$$

$$= (x - 2)^2(x + 3)$$

$$\therefore\ p = 3$$

[3 marks]

(c)



[4 marks]