7.4 Answers

7.4.1 Solution to 7.2 Example

(i)

$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt}\left(8 + \frac{3t^2}{4}\right)$$

$$= \frac{3 \times 2t^1}{4}$$

$$= \frac{3t}{2}$$

When $t = 5$, $a = 7.5 \text{ m s}^{-2}$

[3 marks]

(ii)

$$s = \int v \; dt$$

$$= \int_1^4 8 + \frac{3t^2}{4} \; dt$$

$$= \left[8t + \frac{3t^3}{3 \times 4} \right]_1^4$$

$$= \left[8t + \frac{t^3}{4} \right]_1^4$$

$$= \{ [32 + 16] - [8 + 0.25] \}$$

$$= 48 - 8.25$$

$$= 39.75 \text{ m}$$

[4 marks]

7.4.2 Solution to 7.3 Exercise

Answer 1

$$s = \int v \; dt$$

$$= \int_1^3 6 + 3t^2 \; dt$$

$$= \left[6t + t^3 \right]_1^3$$

$$= [18 + 27] - [6 + 1]$$

$$= 38 \text{ metres}$$

[4 marks]

Answer 2

(i)

$$a = \frac{dv}{dt}$$

$$= 0.72\,t - 0.072\,t^2$$

[3 marks]

(ii)

$$s = \int v \; dt$$

$$= \int_0^{10} 0.36\,t^2 - 0.024\,t^3 \; dt$$

$$= \Big[\, 0.12\,t^3 - 0.006\,t^4 \,\Big]_0^{10}$$

$$= [\, 120 - 60\,] - [\, 0 - 0\,]$$

$$= 60 \text{ metres}$$

[4 marks]

A flaw in this examination question is that the correct answer can be obtained from the incorrect method of

$$s = \left(\frac{u + v}{2} \right) t = \left(\frac{0 + 12}{2} \right) \times 10$$

Answer 3

$$s = \int v \; dt$$

$$= 60 \int_0^{5} t^4 - 10\,t^3 + 25\,t^2 \; dt$$

$$= 60 \times \left[\frac{t^5}{5} - \frac{5\,t^4}{2} + \frac{25\,t^3}{3} \right]_0^5$$

$$= 6250 \text{ metres}$$

[5 marks]

Answer 4

(i)

$$v = 0$$

$$27 - \frac{1}{8}\,t^3 = 0$$

$$\frac{1}{8}\,t^3 = 27$$

$$t^3 = 27 \times 8$$

$$t = 3 \times 2$$

$$= 6 \quad \text{as expected}$$

[1 mark]

(ii)

$$s = \int v \; dt$$

$$= \int_0^6 27 - \frac{1}{8} t^3 \, dt$$

$$= \left[27t - \frac{t^4}{32} \right]_0^6$$

$$= [\,162 - 40.5\,] - [0 - 0\,]$$

$$= 121.5 \text{ metres}$$

[5 marks]

Answer 5
(i)

$$v = \int a \; dt$$

$$= \int 4 - 0.2\,t \; dt$$

$$= 4t - 0.1\,t^2 + c$$

As $v = 0$ when $t = 0$, $c = 0$

$$\therefore \; v = 4t - 0.1\,t^2$$

When $t = 5$, $v = 17.5 \text{ m s}^{-1}$

[3 marks]

(ii) When $t = 20$, $a = 0$
Also, the velocity-time graph is of an inverted parabola, ∩, because of the minus t^2 in the velocity equation.
Thus at $t = 20$, the car is at its maximum velocity.

[1 mark]

(iii)

$$s = \int v \; dt$$

$$= \int_0^{20} 4t - 0.1\,t^2 \, dt$$

$$= \left[2\,t^2 - 0.1 \times \frac{t^3}{3} \right]_0^{20}$$

$$= \left[800 - \frac{800}{3} \right] - [0 - 0\,]$$

$$= 533 \text{ metres}$$

[3 marks]

Answer 6

(a)

$$v = \frac{ds}{dt}$$

$$= 3t^2 + 8t - 5$$

[2 marks]

(b)

$$a = \frac{dv}{dt}$$

$$= 6t + 8$$

Thus, when $t = 2$, $a = 20 \text{ m s}^{-2}$

[2 marks]

Answer 7

(i)

$$s = \int v \; dt$$

$$= \int_0^4 t \, (4 - t) \; dt$$

$$= \int_0^4 4t - t^2 \; dt$$

$$= \left[2t^2 - \frac{t^3}{3} \right]_0^4$$

$$= 32 - \frac{64}{3}$$

$$= 32 - 21\frac{1}{3}$$

$$= 10\frac{2}{3}$$

[3 marks]

(ii) The distance between the lights is $10\frac{2}{3}$ metres.

[1 mark]

(iii)

$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt} \left(4t - t^2 \right)$$

$$= 4 - 2t$$

When $t = 1$, the gradient is 2

[2 marks]

(iv) The acceleration when $t = 1$ second was 2 m s^{-2}

[1 mark]

Answer 8

$$v = \frac{ds}{dt}$$

$$= 48 - 3t^2$$

(i) When $t = 1$, $v = 45$ cm s^{-1}

[2 marks]

(ii) When $t = 0$, $v = 48$ cm s^{-1}

[1 mark]

$$a = \frac{dv}{dt}$$

$$= -6t$$

(iii) When $t = 1$, $a = -6$ cm s^{-2}

[3 marks]

(iv) Come to rest when,

$$v = 0$$

$$48 - 3t^2 = 0$$

$$16 - t^2 = 0$$

$$t = 4 \text{ seconds} \qquad (\text{reject } t = -4)$$

[2 marks]

(v)

$$s = 48t - t^3$$

When $t = 4$, $s = 128$ cm

[2 marks]

(b)

[2 marks]

Answer 9

(a)

$$v = \frac{ds}{dt} = 8t - 2t^2$$

Object is not moving when, $v = 0$

$$8t - 2t^2 = 0$$

$$2t(4 - t) = 0$$

$$\therefore\ v = 0 \text{ when } t = 0 \text{ or } t = 4$$

So, the object started sinking at $t = 0$ and stopped sinking when $t = 4$

[4 marks]

(b) **(i)**

$$a = \frac{dv}{dt} = 8 - 4t$$

When $t = 0$, $a = 8 \text{ m s}^{-2}$ is the initial acceleration

[4 marks]

(ii) Greatest depth will be when $t = 4$ seconds

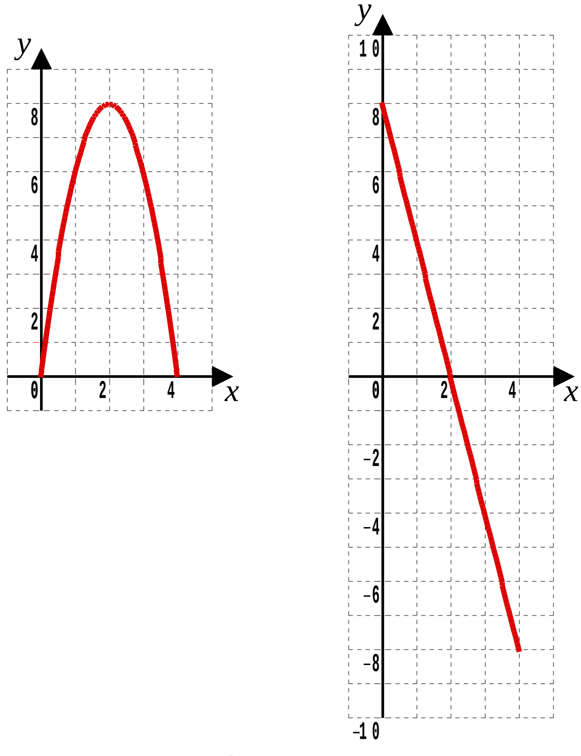
$$s = 4t^2 - \frac{2t^3}{3}$$

$$= 64 - \frac{128}{3}$$

$$= \frac{64}{3} = 21.3 \text{ metres}$$

[2 marks]

(c)



Velocity-time (left) and acceleration-time (right) graphs

[2 marks]

Answer 10

(i)

$$time = \frac{distance}{average\ speed}$$

$$T_{DAY} = \frac{200}{v} \qquad \& \qquad T_{NIGHT} = \frac{200}{v + 20}$$

[2 marks]

(ii)

$$T_{NIGHT} = T_{DAY} - \frac{50}{60}$$

$$\frac{200}{v + 20} = \frac{200}{v} - \frac{5}{6}$$

$$200\,v = 200\,(v + 20) - \frac{5}{6} \times v \times (v + 20)$$

$$1200\,v = 1200\,(v + 20) - 5\,v\,(v + 20)$$

$$1200\,v = 1200\,v + 24000 - 5\,v^2 - 100\,v$$

$$5\,v^2 + 100\,v + 24000 = 0$$

$$v^2 + 20\,v + 4800 = 0 \qquad \text{as expected}$$

[5 marks]

(iii)

$$(v + 80)\,(v - 60) = 0$$

Either $v = -80$ (reject) or $v = 60$ km h^{-1}

Thus, $$T_{DAY} = \frac{200}{60}$$

$$= \frac{10}{3} = 3 \text{ hours and 20 minutes}$$

$$T_{NIGHT} = \frac{200}{60 + 20}$$

$$= \frac{5}{2} = 2 \text{ hours and 30 minutes}$$

[5 marks]

Answer 11

$$distance = average\ speed \times time$$

$$time = \frac{25 \times 400.5}{4.95}$$

$$= 2022.7272... \qquad \text{33 minutes and 43 seconds}$$

[4 marks]