

7.4 Answers

A-Level Pure Mathematics, Year 1 Additional Mathematics Integration I

7.4.1 Solution to 7.2 Example

$$\begin{aligned} \text{(i)} \quad a &= \frac{dv}{dt} \\ &= \frac{d}{dt} \left(8 + \frac{3t^2}{4} \right) \\ &= \frac{3 \times 2 t^1}{4} \\ &= \frac{3t}{2} \end{aligned}$$

$$\text{When } t = 5, a = 7.5 \text{ m s}^{-2}$$

[3 marks]

$$\begin{aligned} \text{(ii)} \quad s &= \int v \, dt \\ &= \int_1^4 8 + \frac{3t^2}{4} \, dt \\ &= \left[8t + \frac{3t^3}{3 \times 4} \right]_1^4 \\ &= \left[8t + \frac{t^3}{4} \right]_1^4 \\ &= \{ [32 + 16] - [8 + 0.25] \} \\ &= 48 - 8.25 \\ &= 39.75 \text{ m} \end{aligned}$$

[4 marks]

7.4.2 Solution to 7.3 Exercise

Answer 1

$$\begin{aligned} s &= \int v \, dt \\ &= \int_1^3 6 + 3t^2 \, dt \\ &= \left[6t + t^3 \right]_1^3 \\ &= [18 + 27] - [6 + 1] \\ &= 38 \text{ metres} \end{aligned}$$

[4 marks]

Answer 2

$$\begin{aligned}
 \text{(i)} \quad a &= \frac{dv}{dt} \\
 &= 0.72t - 0.072t^2
 \end{aligned}$$

[3 marks]

$$\begin{aligned}
 \text{(ii)} \quad s &= \int v \, dt \\
 &= \int_0^{10} 0.36t^2 - 0.024t^3 \, dt \\
 &= \left[0.12t^3 - 0.006t^4 \right]_0^{10} \\
 &= [120 - 60] - [0 - 0] \\
 &= 60 \text{ metres}
 \end{aligned}$$

[4 marks]

A flaw in this examination question is that the correct answer can be obtained from the incorrect method of

$$s = \left(\frac{u + v}{2} \right) t = \left(\frac{0 + 12}{2} \right) \times 10$$

Answer 3

$$\begin{aligned}
 s &= \int v \, dt \\
 &= 60 \int_0^5 t^4 - 10t^3 + 25t^2 \, dt \\
 &= 60 \times \left[\frac{t^5}{5} - \frac{5t^4}{2} + \frac{25t^3}{3} \right]_0^5 \\
 &= 6250 \text{ metres}
 \end{aligned}$$

[5 marks]**Answer 4**

$$\begin{aligned}
 \text{(i)} \quad v &= 0 \\
 27 - \frac{1}{8}t^3 &= 0 \\
 \frac{1}{8}t^3 &= 27 \\
 t^3 &= 27 \times 8 \\
 t &= 3 \times 2 \\
 &= 6 \quad \text{as expected}
 \end{aligned}$$

[1 mark]

(ii)

$$\begin{aligned} s &= \int v \, dt \\ &= \int_0^6 27 - \frac{1}{8} t^3 \, dt \\ &= \left[27t - \frac{t^4}{32} \right]_0^6 \\ &= [162 - 40.5] - [0 - 0] \\ &= 121.5 \text{ metres} \end{aligned}$$

[5 marks]

Answer 5

(i)

$$\begin{aligned} v &= \int a \, dt \\ &= \int 4 - 0.2 t \, dt \\ &= 4t - 0.1 t^2 + c \end{aligned}$$

As $v = 0$ when $t = 0$, $c = 0$

$$\therefore v = 4t - 0.1 t^2$$

When $t = 5$, $v = 17.5 \text{ m s}^{-1}$

[3 marks]

(ii) When $t = 20$, $a = 0$

Also, the velocity-time graph is of an inverted parabola, $\cap$, because of the minus t^2 in the velocity equation.

Thus at $t = 20$, the car is at its maximum velocity.

[1 mark]

(iii)

$$\begin{aligned} s &= \int v \, dt \\ &= \int_0^{20} 4t - 0.1 t^2 \, dt \\ &= \left[2 t^2 - 0.1 \times \frac{t^3}{3} \right]_0^{20} \\ &= \left[800 - \frac{800}{3} \right] - [0 - 0] \\ &= 533 \text{ metres} \end{aligned}$$

[3 marks]

Answer 6

$$\begin{aligned}
 \text{(a)} \quad v &= \frac{ds}{dt} \\
 &= 3t^2 + 8t - 5
 \end{aligned}$$

[2 marks]

$$\begin{aligned}
 \text{(b)} \quad a &= \frac{dv}{dt} \\
 &= 6t + 8
 \end{aligned}$$

Thus, when $t = 2$, $a = 20 \text{ m s}^{-2}$ **[2 marks]****Answer 7****(i)**

$$\begin{aligned}
 s &= \int v \, dt \\
 &= \int_0^4 t (4 - t) \, dt \\
 &= \int_0^4 4t - t^2 \, dt \\
 &= \left[2t^2 - \frac{t^3}{3} \right]_0^4 \\
 &= 32 - \frac{64}{3} \\
 &= 32 - 21\frac{1}{3} \\
 &= 10\frac{2}{3}
 \end{aligned}$$

[3 marks]**(ii)** The distance between the lights is $10\frac{2}{3}$ metres.**[1 mark]****(iii)**

$$\begin{aligned}
 a &= \frac{dv}{dt} \\
 &= \frac{d}{dt} (4t - t^2) \\
 &= 4 - 2t
 \end{aligned}$$

When $t = 1$, the gradient is 2**[2 marks]****(iv)** The acceleration when $t = 1$ second was 2 m s^{-2} **[1 mark]**

Answer 8

$$v = \frac{ds}{dt}$$
$$= 48 - 3t^2$$

(i) When $t = 1$, $v = 45 \text{ cm s}^{-1}$

[2 marks]

(ii) When $t = 0$, $v = 48 \text{ cm s}^{-1}$

[1 mark]

$$a = \frac{dv}{dt}$$
$$= -6t$$

(iii) When $t = 1$, $a = -6 \text{ cm s}^{-2}$

[3 marks]

(iv) Come to rest when,

$$v = 0$$
$$48 - 3t^2 = 0$$
$$16 - t^2 = 0$$
$$t = 4 \text{ seconds} \quad (\text{reject } t = -4)$$

[2 marks]

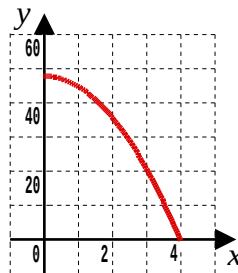
(v)

$$s = 48t - t^3$$

When $t = 4$, $s = 128 \text{ cm}$

[2 marks]

(b)



[2 marks]

Answer 9

(a)
$$v = \frac{ds}{dt} = 8t - 2t^2$$

Object is not moving when, $v = 0$

$$8t - 2t^2 = 0$$

$$2t(4 - t) = 0$$

$$\therefore v = 0 \text{ when } t = 0 \text{ or } t = 4$$

So, the object started sinking at $t = 0$ and stopped sinking when $t = 4$

[4 marks]

(b) (i)
$$a = \frac{dv}{dt} = 8 - 4t$$

When $t = 0$, $a = 8 \text{ m s}^{-2}$ is the initial acceleration

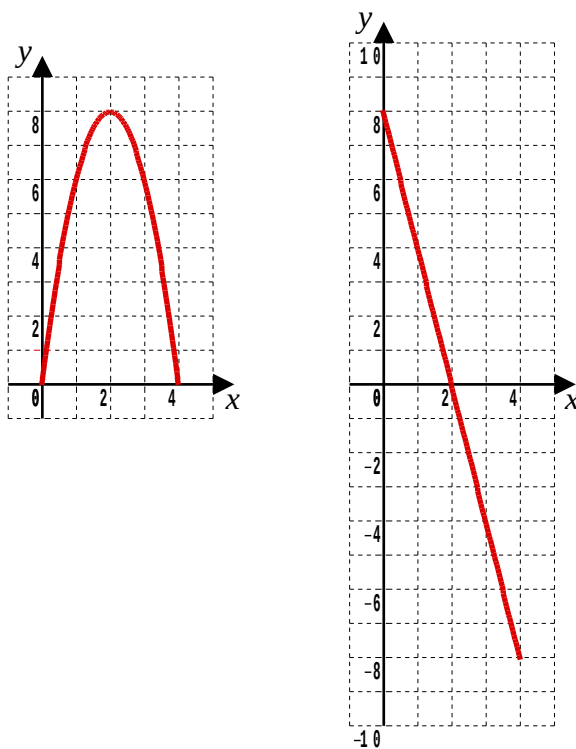
[4 marks]

(ii) Greatest depth will be when $t = 4$ seconds

$$\begin{aligned} s &= 4t^2 - \frac{2t^3}{3} \\ &= 64 - \frac{128}{3} \\ &= \frac{64}{3} = 21.3 \text{ metres} \end{aligned}$$

[2 marks]

(c)



Velocity-time (left) and acceleration-time (right) graphs

[2 marks]

Answer 10

(i)
$$time = \frac{distance}{average\ speed}$$

$$T_{DAY} = \frac{200}{v} \quad \& \quad T_{NIGHT} = \frac{200}{v + 20}$$

[2 marks]

(ii)
$$T_{NIGHT} = T_{DAY} - \frac{50}{60}$$

$$\frac{200}{v + 20} = \frac{200}{v} - \frac{5}{6}$$

$$200v = 200(v + 20) - \frac{5}{6} \times v \times (v + 20)$$

$$1200v = 1200(v + 20) - 5v(v + 20)$$

$$1200v = 1200v + 24000 - 5v^2 - 100v$$

$$5v^2 + 100v + 24000 = 0$$

$$v^2 + 20v + 4800 = 0 \quad \text{as expected}$$

[5 marks]

(iii)
$$(v + 80)(v - 60) = 0$$

Either $v = -80$ (reject) or $v = 60 \text{ km h}^{-1}$

Thus,
$$T_{DAY} = \frac{200}{60}$$

$$= \frac{10}{3} = 3 \text{ hours and } 20 \text{ minutes}$$

$$T_{NIGHT} = \frac{200}{60 + 20}$$

$$= \frac{5}{2} = 2 \text{ hours and } 30 \text{ minutes}$$

[5 marks]**Answer 11**

$$distance = average\ speed \times time$$

$$time = \frac{25 \times 400.5}{4.95}$$

$$= 2022.7272... \quad 33 \text{ minutes and } 43 \text{ seconds}$$

[4 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

© 2025 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com