8.2 Answers

A-Level Pure Mathematics, Year 1
Additional Mathematics
Integration I

Answer 1

$$\int_1^3 12x^2 - 2 \; dx = \left[4x^3 - 2x \right]_1^3$$

$$= (4 \times 27 - 2 \times 3) - (4 \times 1 - 2 \times 1)$$

$$= 102 - 2$$

$$= 100 \qquad \square$$

[4 marks]

Answer 2

$$\int \frac{9x^5}{2} \, dx = \frac{9x^6}{2 \times 6} + c$$

$$= \frac{3x^6}{4} + c$$

[3 marks]

Answer 3

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5) \; dx = 2x^4 + 4x^{\frac{3}{2}} - 5x + c$$

[4 marks]

Answer 4

$$y = 2x^3 + \frac{3}{x^2}$$

$$= 2x^3 + 3x^{-2}$$

(a)

$$\frac{dy}{dx} = 6x^2 - 6x^{-3}$$

[3 marks]

(b)

$$\int y \, dx = \frac{x^4}{2} - 3x^{-1} + c$$

$$= \frac{x^4}{2} - \frac{3}{x} + c$$

[3 marks]

Answer 5

$$y = \int \frac{dy}{dx}\, dx$$

$$= \int 2x + 2 \; dx$$

$$= x^2 + 2x + c$$

Told that this curve passes through (3, 0)

$$0 = 3^2 + 2 \times 3 + c$$

$$0 = 15 + c$$

$$c = -15$$

Thus, the equation of the curve is

$$y = x^2 + 2x - 15$$

[5 marks]

Answer 6

$$\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2} \right) dx = \int_1^2 \left(3x^2 + 5 + 4x^{-2} \right) dx$$

$$= \left[x^3 + 5x - 4x^{-1} \right]_1^2$$

$$= \left[x^3 + 5x - \frac{4}{x} \right]_1^2$$

$$= (8 + 10 - 2) - (1 + 5 - 4)$$

$$= 16 - 2$$

$$= 14$$

[5 marks]

Answer 7

Area of the lawn is given by,

$$\int_0^8 8x - x^2 \; dx = \left[4x^2 - \frac{x^3}{3} \right]_0^8$$

$$= \left[4 \times 8^2 - \frac{8^3}{3} \right]$$

$$= \frac{256}{3} \text{ m}^2$$

Area whole garden is $20 \times 8 = 160$ m^2

Percentage that's lawn is $\frac{256}{3 \times 160} \times 100 = 53\frac{1}{3}$ □

[6 marks]

Answer 8

(a) To find the points of intersection of line and curve,

$$10 - x = 10x - x^2 - 8$$

$$x^2 - 11x + 18 = 0$$

$$(x - 2)(x - 9) = 0$$

$$\text{Either } x = 2 \text{ or } x = 9$$

$$\text{That is, } A(2,\ 8),\ B(9,\ 1)$$

[5 marks]

(b)

$$Area\ R = \int_2^9 10x - x^2 - 8\ dx - \int_2^9 10 - x\ dx$$

$$= \int_2^9 11x - x^2 - 18\ dx$$

$$= \left[\frac{11x^2}{2} - \frac{x^3}{3} - 18x \right]_2^9$$

$$= \left[\frac{891}{2} - 243 - 162 \right] - \left[22 - \frac{8}{3} - 36 \right]$$

$$= \left[\frac{81}{2} \right] - \left[-\frac{50}{3} \right]$$

$$= \frac{343}{6} = 57\frac{1}{6}$$

[7 marks]

Answer 9

$$\int_0^1 x(x-1)(x-5)\,dx = \int_0^1 x^3 - 6x^2 + 5x\,dx$$

$$= \left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_0^1$$

$$= \left[\frac{1}{4} - 2 + \frac{5}{2}\right]$$

$$= \frac{3}{4}$$

$$\int_1^2 x(x-1)(x-5)\,dx = \int_1^2 x^3 - 6x^2 + 5x\,dx$$

$$= \left[\frac{x^4}{4} - 2x^3 + \frac{5x^2}{2}\right]_1^2$$

$$= [4 - 16 + 10] - \left[\frac{1}{4} - 2 + \frac{5}{2}\right]$$

$$= -2 - \frac{3}{4}$$

$$= -\frac{11}{4}$$

$$= -2\frac{3}{4}$$

Thus, the area shaded will be,

$$\frac{3}{4} + \frac{11}{4} = \frac{7}{2}$$

[9 marks]

Answer 10

(i)

$$time = \frac{distance}{speed}$$

$$time_{GAVIN} = \frac{140}{v} \qquad time_{SIMON} = \frac{140}{v+5}$$

[2 marks]

(ii) As 15 minutes is quarter of an hour,

Gavin's time minus Simon's time will be one quarter

$$\frac{140}{v} - \frac{140}{v+5} = \frac{1}{4}$$

Multiply through by $4v(v+5)$

$$4(v+5)140 - 4v\,140 = v(v+5)$$

$$560v + 2800 - 560v = v^2 + 5v$$

$$v^2 + 5v - 2800 = 0$$

[5 marks]

(iii)

$$v = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times (-2800)}}{2 \times 1}$$

$$= \frac{-5 \pm \sqrt{25 + 11200}}{2}$$

$$= 50.474, \quad \text{(reject } -55.474\text{)}$$

$$\therefore\ time_{GAVIN} = \frac{140}{50.474} = \text{2 hours 46 minutes}$$

$$time_{SIMON} = \frac{140}{55.474} = \text{2 hours 31 minutes}$$

[5 marks]