2.3 Answers

2.3.1 Solution to 2.1 Example (Teaching Video)

When a mathematician writes $cos^2 x$ they actually mean $(cos\, x)^2$
Unfortunately, this sloppy practice is widespread and you need to learn to live with it.
So,

$$4\, cos^2 x + cos\, x - 3 = 0 \qquad \text{for} \quad 0° \leqslant x \leqslant 360°$$

is better written as,

$$4\, (cos\, x)^2 + cos\, x - 3 = 0 \qquad \text{for} \quad 0° \leqslant x \leqslant 360°$$

which makes it more obvious that this is really a quadratic equation, but in disguise.

Let $z = cos\, x$ to get,

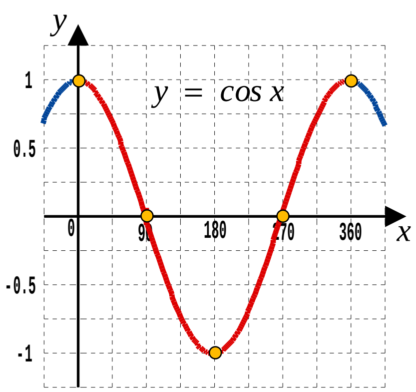
$$4 z^2 + z - 3 = 0$$

$$(4z - 3) (z + 1) = 0$$

$$\text{Either } 4z - 3 = 0 \text{ or } z + 1 = 0$$

$$z = \frac{3}{4} \text{ or } z = -1$$

$$cos\, x = \frac{3}{4} \text{ or } cos\, x = -1$$



For $cos\, x = -1$ the answer is read off from the sketch graph as $x = 180°$

$$\text{For } cos\, x = \frac{3}{4} \Rightarrow x = arccos\left(\frac{3}{4}\right)$$

$$= 41.4,\ 360 - 41.4$$

$$= 41.4,\ 318.6$$

The three solutions to the original equation are $41.4°,\ 180°,\ 318.6°$

[6 marks]

2.3.2 Solutions to 2.2 Exercise

Answer 1

(i) From the graph it would seem that there is one solution around 30°, another around 150°, and possibly one or two more close to 270° although it's not clear if the curve gets to or crosses the *x*-axis near 270°.

[4 marks]

(ii) $2\,(\sin x)^2 + \sin x - 1 = 0 \quad \text{for} \quad 0° \leqslant x \leqslant 360°$

Let $z = \sin x$ to get,

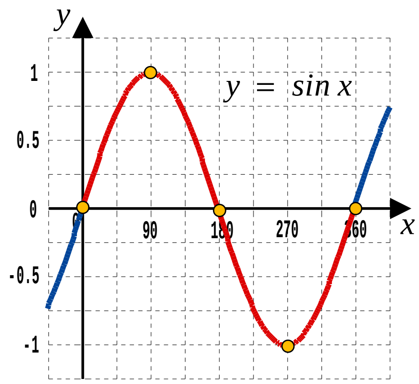
$$2\,z^2 + z - 1 = 0$$

$$(2z - 1)(z + 1) = 0$$

$$\text{Either } 2z - 1 = 0 \quad \text{or} \quad z + 1 = 0$$

$$z = \frac{1}{2} \quad \text{or} \quad z = -1$$

$$\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = -1$$



For $\sin x = -1$ the answer is read off from the sketch graph as $x = 270°$

$$\text{For } \sin x = \frac{1}{2} \Rightarrow x = \arcsin\left(\frac{1}{2}\right)$$

$$= 30,\ 180 - 30$$

$$= 30,\ 150$$

The three solutions to the original equation are $30°,\ 150°,\ 270°$

[6 marks]

Answer 2

(i) From the graph it would seem that there is one solution around 60°, another around 300°, and possibly one or two more close to 0° and likewise close to 360° although it's not clear if the curve gets to or crosses the *x*-axis near 0° and 360°

[4 marks]

(ii) $2(\cos x)^2 - 3\cos x + 1 = 0$ for $0° \leqslant x \leqslant 360°$

Let $z = \cos x$ to get,

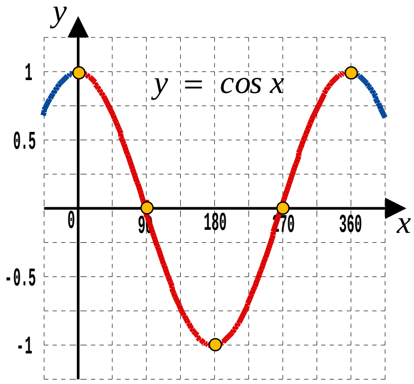
$$2z^2 - 3z + 1 = 0$$

$$(2z - 1)(z - 1) = 0$$

$$\text{Either } 2z - 1 = 0 \quad \text{or} \quad z - 1 = 0$$

$$z = \frac{1}{2} \quad \text{or} \quad z = 1$$

$$\cos x = \frac{1}{2} \quad \text{or} \quad \cos x = 1$$



For $\cos x = 1$ the answers read off from the sketch graph are $x = 0°,\ 360°$

$$\text{For } \cos x = \frac{1}{2} \Rightarrow x = \arccos\left(\frac{1}{2}\right)$$

$$= 60,\ 360 - 60$$

$$= 60,\ 300$$

The four solutions to the original equation are $0°,\ 60°,\ 300°,\ 360°$

[6 marks]

Answer 3

$$15\,(\sin x)^2 - 11\sin x + 2 = 0$$

Let $z = \sin x$ to get,

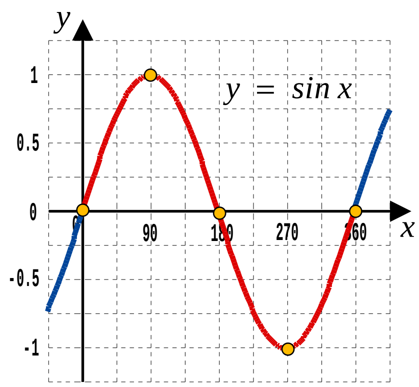
$$15z^2 - 11z + 2 = 0$$

$$(3z - 1)(5z - 2) = 0$$

Either $3z - 1 = 0$ or $5z - 2 = 0$

$$z = \frac{1}{3} \quad \text{or} \quad z = \frac{2}{5}$$

$$\sin x = \frac{1}{3} \quad \text{or} \quad \sin x = \frac{2}{5}$$



$x = \arcsin\left(\frac{1}{3}\right)$ or $x = \arcsin\left(\frac{2}{5}\right)$

$x = 19.5°,\ 180 - 19.5$ or $x = 23.6,\ 180 - 23.6$

$$\therefore\ x = 19.5°,\ 23.6°,\ 156.4°,\ 160.5°$$

[10 marks]