

2.3 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Trigonometry IV

2.3.1 Solution to 2.1 Example (Teaching Video)

When a mathematician writes $\cos^2 x$ they actually mean $(\cos x)^2$

Unfortunately, this sloppy practice is widespread and you need to learn to live with it.

So,

$$4 \cos^2 x + \cos x - 3 = 0 \quad \text{for } 0^\circ \leq x \leq 360^\circ$$

is better written as,

$$4(\cos x)^2 + \cos x - 3 = 0 \quad \text{for } 0^\circ \leq x \leq 360^\circ$$

which makes it more obvious that this is really a quadratic equation, but in disguise.

Let $z = \cos x$ to get,

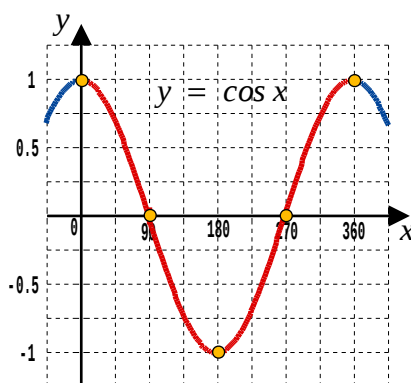
$$4z^2 + z - 3 = 0$$

$$(4z - 3)(z + 1) = 0$$

$$\text{Either } 4z - 3 = 0 \text{ or } z + 1 = 0$$

$$z = \frac{3}{4} \text{ or } z = -1$$

$$\cos x = \frac{3}{4} \text{ or } \cos x = -1$$



For $\cos x = -1$ the answer is read off from the sketch graph as $x = 180^\circ$

$$\begin{aligned} \text{For } \cos x = \frac{3}{4} &\Rightarrow x = \arccos\left(\frac{3}{4}\right) \\ &= 41.4, 360 - 41.4 \\ &= 41.4, 318.6 \end{aligned}$$

The three solutions to the original equation are 41.4° , 180° , 318.6°

[6 marks]

2.3.2 Solutions to 2.2 Exercise

Answer 1

- (i) From the graph it would seem that there is one solution around 30° , another around 150° , and possibly one or two more close to 270° although it's not clear if the curve gets to or crosses the x-axis near 270° .

[4 marks]

- (ii) $2(\sin x)^2 + \sin x - 1 = 0$ for $0^\circ \leq x \leq 360^\circ$

Let $z = \sin x$ to get,

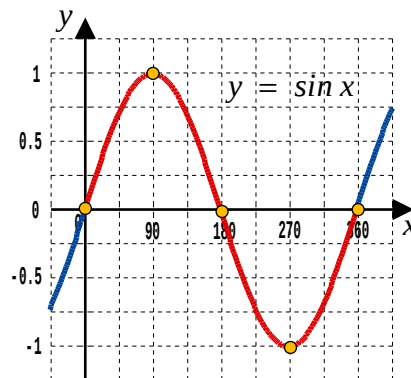
$$2z^2 + z - 1 = 0$$

$$(2z - 1)(z + 1) = 0$$

$$\text{Either } 2z - 1 = 0 \text{ or } z + 1 = 0$$

$$z = \frac{1}{2} \text{ or } z = -1$$

$$\sin x = \frac{1}{2} \text{ or } \sin x = -1$$



For $\sin x = -1$ the answer is read off from the sketch graph as $x = 270^\circ$

$$\begin{aligned} \text{For } \sin x = \frac{1}{2} &\Rightarrow x = \arcsin\left(\frac{1}{2}\right) \\ &= 30, 180 - 30 \\ &= 30, 150 \end{aligned}$$

The three solutions to the original equation are 30° , 150° , 270°

[6 marks]

Answer 2

- (i) From the graph it would seem that there is one solution around 60° , another around 300° , and possibly one or two more close to 0° and likewise close to 360° although it's not clear if the curve gets to or crosses the x-axis near 0° and 360°

[4 marks]

(ii) $2(\cos x)^2 - 3\cos x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$

Let $z = \cos x$ to get,

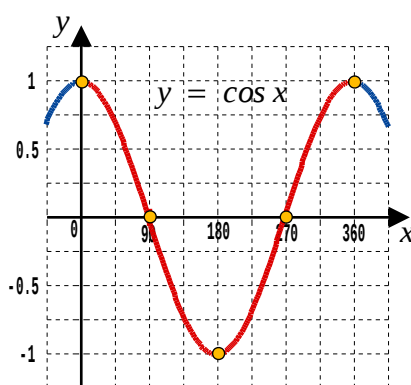
$$2z^2 - 3z + 1 = 0$$

$$(2z - 1)(z - 1) = 0$$

Either $2z - 1 = 0$ or $z - 1 = 0$

$$z = \frac{1}{2} \text{ or } z = 1$$

$$\cos x = \frac{1}{2} \text{ or } \cos x = 1$$



For $\cos x = 1$ the answers read off from the sketch graph are $x = 0^\circ, 360^\circ$

$$\begin{aligned} \text{For } \cos x = \frac{1}{2} &\Rightarrow x = \arccos\left(\frac{1}{2}\right) \\ &= 60, 360 - 60 \\ &= 60, 300 \end{aligned}$$

The four solutions to the original equation are $0^\circ, 60^\circ, 300^\circ, 360^\circ$

[6 marks]

Answer 3

$$15 (\sin x)^2 - 11 \sin x + 2 = 0$$

Let $z = \sin x$ to get,

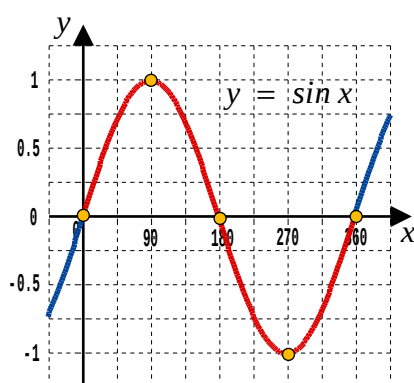
$$15z^2 - 11z + 2 = 0$$

$$(3z - 1)(5z - 2) = 0$$

Either $3z - 1 = 0$ or $5z - 2 = 0$

$$z = \frac{1}{3} \text{ or } z = \frac{2}{5}$$

$$\sin x = \frac{1}{3} \text{ or } \sin x = \frac{2}{5}$$



$$x = \arcsin\left(\frac{1}{3}\right) \quad \text{or} \quad x = \arcsin\left(\frac{2}{5}\right)$$

$$x = 19.5^\circ, 180 - 19.5 \quad \text{or} \quad x = 23.6^\circ, 180 - 23.6$$

$$\therefore x = 19.5^\circ, 23.6^\circ, 156.4^\circ, 160.5^\circ$$

[10 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com