3.3 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Trigonometry IV

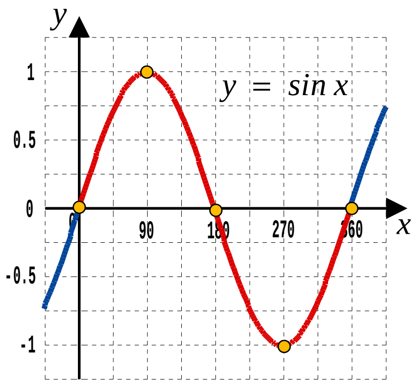
3.3.1 Solution to 3.1 Teaching Video

x	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

3.3.2 Solutions to 3.2 Exercise

Answer 1

(i)



[2 marks]

(ii)

x	0	30	45	60	90	120	135	150
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$

x	180	210	225	240	270	300	315	330
$\sin x$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$

[6 marks]

Answer 2

$$4(\sin x)^2 + 4\sin + 1 = 0 \qquad \text{for} \quad 0° \leqslant x \leqslant 360°$$

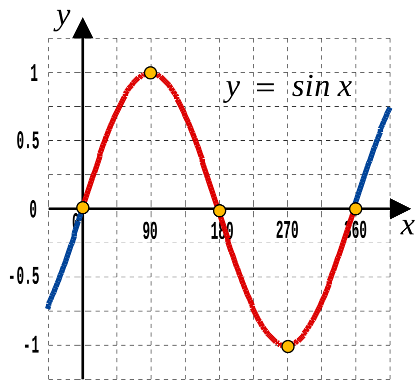
Let $z = \sin x$ to get,

$$4z^2 + 4z + 1 = 0$$
$$(2z + 1)(2z + 1) = 0$$
$$\therefore \; 2z + 1 = 0$$
$$z = -\frac{1}{2}$$
$$\sin x = -\frac{1}{2}$$

$$\text{For } \sin x = +\frac{1}{2} \Rightarrow x = \arcsin\left(\frac{1}{2}\right)$$
$$= 30 \leftarrow \text{ Working Angle}$$

The solutions sought in the range $0° \leqslant x \leqslant 360°$ will be two of

x $\qquad$ $180 - x$ $\qquad$ $180 + x$ $\qquad$ or $\qquad$ $360 - x$



From the $y = \sin x$ sketch graph, solution sought are 180 + 30 and 360 – 30

$$\therefore \; x = 210°, \;\; 330°$$

Alternatively, these could have been grabbed from the table made in answer to question 1

[6 marks]

Answer 3

(i) The graph does not cross the x-axis over the domain of $0° \leqslant x \leqslant 360°$
That is, there is no value of x for which the function has no height.

The equation $3\sin^2 x + \sin x + 1 = 0$ will have no real solutions.

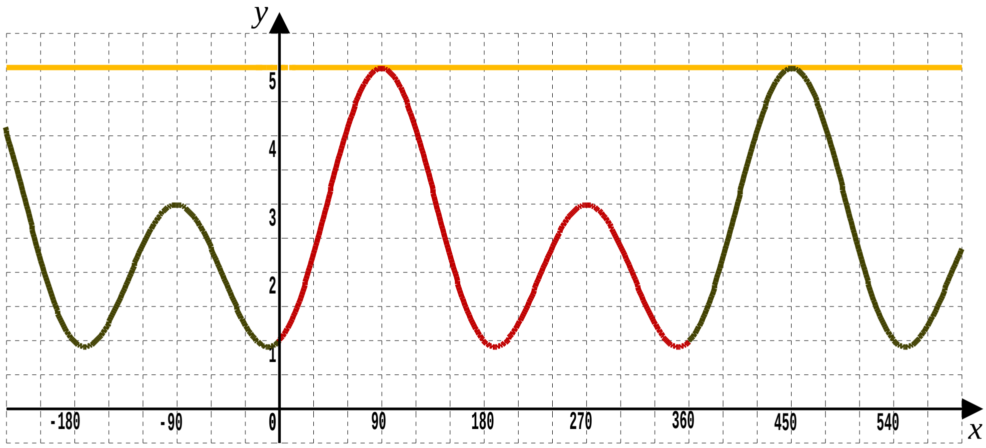
$\therefore$ Trying to find solutions is indeed futile.

[2 marks]

(ii) Over the expanded domain of $-180° \leqslant x \leqslant 540°$ the equation will still have no real solutions. This is because the function is made up of further copies of that already considered between $0° \leqslant x \leqslant 360°$. to both the left and the right of that interval.

[2 marks]

(iii) Here is the graph with the horizontal straight line $y = 5$ added,



The straight line intersects the curve at about 90°, and so that is the probable solution to

$$3\sin^2 x + \sin x + 1 = 5 \qquad \text{for } 0° \leqslant x \leqslant 360°$$

[2 marks]

(iv)

$$3\sin^2 x + \sin x + 1 = 5$$

$$3\sin^2 x + \sin x - 4 = 0$$

Let $z = \sin x$ to get,

$$3z^2 + z - 4 = 0$$

$$(3z + 4)(z - 1) = 0$$

$$\text{Either } 3z + 4 = 0 \text{ or } z - 1 = 0$$

$$z = -\frac{4}{3} \text{ or } z = 1$$

$\sin x = -\dfrac{4}{3}$ has no solutions because that's less than it's minimum value

$\sin x = 1$ gives $x = 90°$ as the only solution

[6 marks]

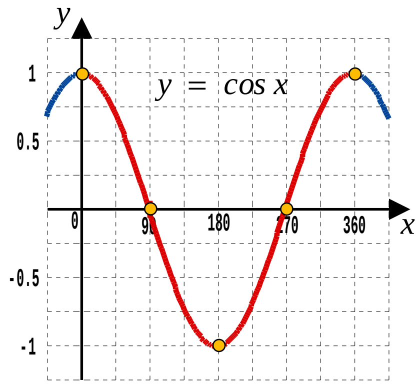
Answer 4

$$\left(2\cos x - \sqrt{3} \right)\left(2\cos x + \sqrt{2} \right) = 0$$

Either $\left(2\cos x - \sqrt{3} \right) = 0$ or $\left(2\cos x + \sqrt{2} \right) = 0$

$$\cos x = \frac{\sqrt{3}}{2} \quad \text{or} \quad \cos x = -\frac{\sqrt{2}}{2}$$

For $\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \arccos\left(\frac{\sqrt{3}}{2}\right) \Rightarrow x = 30° \leftarrow$ working angle

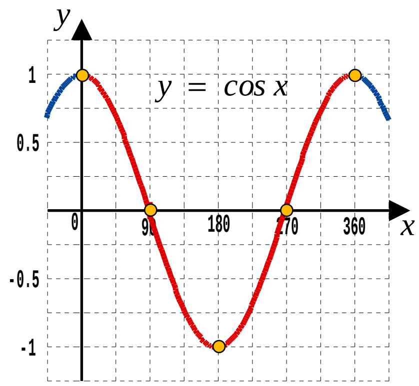


The solutions sought in the range $0° \leqslant x \leqslant 360°$ will be two of

x $\quad$ $180 - x$ $\quad$ $180 + x$ $\quad$ or $\quad$ $360 - x$

$$\therefore \ x = 30° \quad \text{or} \quad x = 330°$$

For $\cos x = +\frac{\sqrt{2}}{2} \Rightarrow x = \arccos\left(\frac{\sqrt{2}}{2}\right) \Rightarrow x = 45° \leftarrow$ working angle



The solutions sought in the range $0° \leqslant x \leqslant 360°$ will be two of

x $\quad$ $180 - x$ $\quad$ $180 + x$ $\quad$ or $\quad$ $360 - x$

$$\therefore \ x = 135° \quad \text{or} \quad x = 225°$$

Thus, in total there are four solutions; $30°, \ 135°, \ 225°, \ 330°$

[6 marks]