4.4 Answers

4.4.1 Solution to 4.2 Example (Teaching Video)

$$5\sin x = 2\cos^2 x - 4$$

$$5\sin x = 2(1 - \sin^2 x) - 4$$

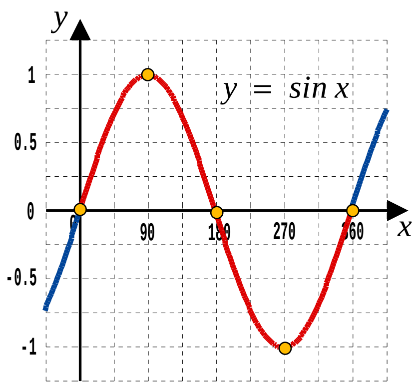
$$5\sin x = 2 - 2\sin^2 x - 4$$

$$2\sin^2 x + 5\sin x + 2 = 0$$

$$(2\sin x + 1)(\sin x + 2) = 0$$

Either $2\sin x + 1 = 0$ or $\sin x + 2 = 0$

$$\sin x = -\frac{1}{2} \qquad \sin x = -2$$



$x = \arcsin\left(\frac{1}{2}\right)$ $\qquad$ $x = \arcsin(-2)$

$x = 30°$ (working angle) $\qquad$ No solutions

Solutions are $\{180 + 30,\ 360 - 30\}$

i.e. $\{210°,\ 330°\}$

[6 marks]

4.4.2 Solutions to 4.3 Exercise

Answer 1

(a)

$$5 \sin x = 1 + 2 \cos^2 x$$

$$5 \sin x = 1 + 2\left(1 - \sin^2 x\right)$$

$$5 \sin x = 1 + 2 - 2 \sin^2 x$$

$$2 \sin^2 x + 5 \sin x - 3 = 0 \qquad \square$$

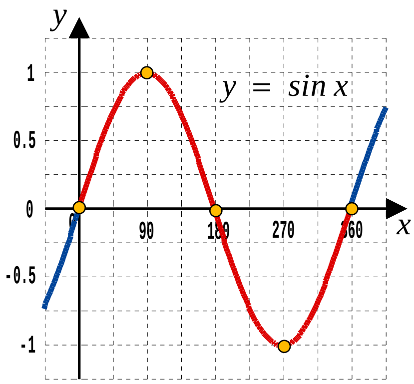
[2 marks]

(b)

$$(2 \sin x - 1) (\sin x + 3) = 0$$

Either $2 \sin x - 1 = 0$ or $\sin x + 3 = 0$

$$\sin x = \frac{1}{2} \qquad\qquad \sin x = -3$$



$x = \arcsin\left(\frac{1}{2}\right)$ $\qquad\qquad$ $x = \arcsin(-3)$

$x = 30°$ (working angle) $\qquad\qquad$ No solutions

$$\therefore \quad x \in \{ 30°, \ 150° \}$$

[4 marks]

Answer 2

$$\cos^2 x - 3 \cos x = \sin^2 x + 4$$

$$\cos^2 x - 3 \cos x = \left(1 - \cos^2 x\right) + 4$$

$$2 \cos^2 x - 3 \cos x - 5 = 0$$

$$(2 \cos x - 5)(\cos x + 1) = 0$$

Either $2 \cos x - 5 = 0$ or $\cos x = -1$

But $\cos x = \frac{5}{2}$ has no solutions as the max value of the *cosine* function is 1

The only solution come from $\cos x = -1 \Rightarrow x = 180°$

[6 marks]

Answer 3

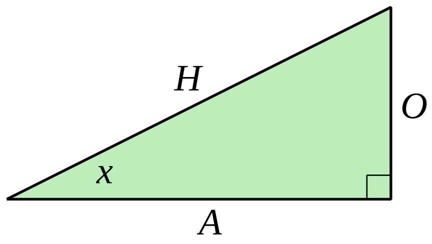
Another Key Trigonometric Identity

$$\frac{\sin x}{\cos x} = \tan x \qquad (\text{ Provided } \cos x \neq 0 \text{ })$$

Proof

Consider a right angled triangle with sides labelled;

O for Opposite angle *x*
A for Adjacent to angle *x*
H for Hypotenuse



$$\begin{aligned}
\text{LHS} &= \frac{\sin x}{\cos x} \\
&= \sin x \div \cos x \\
&= \frac{O}{H} \div \frac{A}{H} \\
&= \frac{O}{H} \times \frac{H}{A} \\
&= \frac{O}{A} \\
&= \tan x \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

Answer 4

(i)

$$\text{LHS} = \frac{1 - cos^2 x}{1 - sin^2 x}$$

$$= \frac{sin^2 x}{cos^2 x}$$

$$= tan^2 x$$

$$= \text{RHS} \qquad \square$$

[1 mark]

(ii)

$$\frac{1 - cos^2 x}{1 - sin^2 x} = 3 - 2\,tan\,x$$

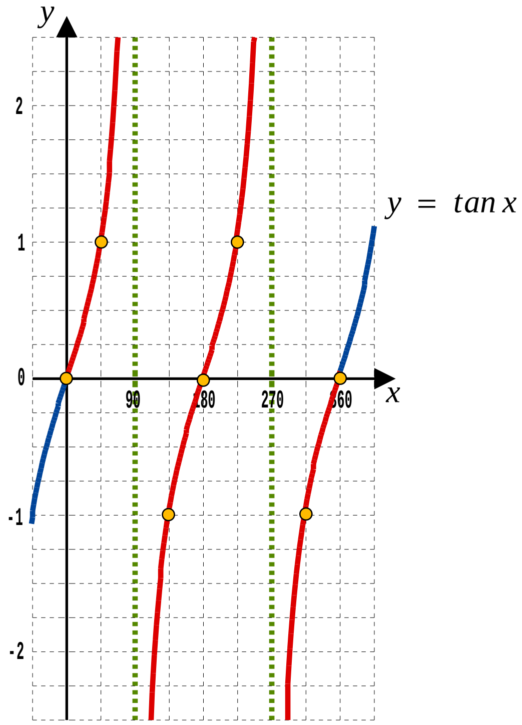
$$tan^2 x = 3 - 2\,tan\,x$$

$$tan^2 x + 2\,tan\,x - 3 = 0$$

$$(\,tan\,x + 3\,)(\,tanx - 1\,) = 0$$

Either $tan\,x + 3 = 0$ or $tanx - 1 = 0$

$tan\,x = -3$ or $tan\,x = 1$



Note that we're only asked for answers over the domain $0 \leqslant x \leqslant 180°$

$tan\,x = 3 \Rightarrow x = arctan\,(3) \Rightarrow x = 71.6°$ working angle

Thus for $tan\,x = -3$ we want $180 - 71.6 = 108.4°$

$tan\,x = 1 \Rightarrow x = arctan\,(1) \Rightarrow x = 45°$

So, two answers, $x \in \{\,45°,\ 108.4°\,\}$

[4 marks]

Answer 5

(a)

$$8\tan x = -3\cos x$$

$$8 \times \frac{\sin x}{\cos x} = -3\cos x$$

Multiply both sides by $\cos x$

$$8\sin x = -3\cos^2 x$$

$$8\sin x = -3\left(1 - \sin^2 x\right)$$

$$8\sin x = -3 + 3\sin^2 x$$

$$3\sin^2 x - 8\sin x - 3 = 0 \qquad \square$$

[3 marks]

(b)

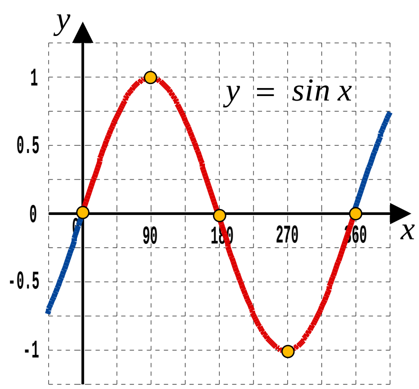
$$8\tan x = -3\cos x$$

$$3\sin^2 x - 8\sin x - 3 = 0 \qquad \text{from part (a)}$$

$$(3\sin x + 1)(\sin x - 3) = 0$$

Either $3\sin x + 1 = 0$ or $\sin x - 3 = 0$

$$\sin x = -\frac{1}{3} \quad \text{or} \quad \sin x = 3 \text{ (No solutions)}$$



Solving $\sin x = +\frac{1}{3} \Rightarrow x = \arcsin\left(\frac{1}{3}\right) \Rightarrow x = 19.5° \leftarrow$ working angle

$\therefore$ For $\sin x = -\frac{1}{3}$ we'll have $x = 199.5°,\ 340.5°$

So, two answers, $x \in \{\, 199.5°,\ 340.5° \,\}$

[5 marks]

Answer 6

(a)

$$5\cos^2 x = 3(1 + \sin x)$$

$$5(1 - \sin^2 x) = 3 + 3\sin x$$

$$5 - 5\sin^2 x = 3 + 3\sin x$$

$$5\sin^2 x + 3\sin x - 2 = 0 \qquad \square$$

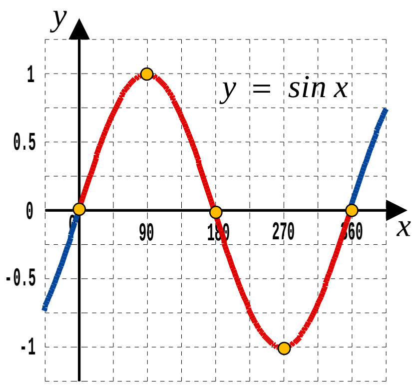
[2 marks]

(b)

$$(5\sin x - 2)(\sin x + 1) = 0$$

Either $5\sin x - 2 = 0$ or $\sin x + 1 = 0$

$$\sin x = \frac{2}{5} \quad \text{or} \quad \sin x = -1$$



$\sin x = 0.4$ gives $23.6°, \ 156.4°$

$\sin x = -1$ gives $270°$

So, three answers, $x \in \{ 23.6°, \ 156.4°, \ 270° \}$

[5 marks]

Answer 7

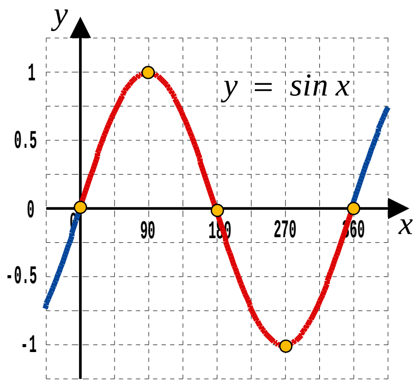
(a)

$$3\sin^2 x + 7\sin x = \cos^2 x - 4$$
$$3\sin^2 x + 7\sin x = \left(1 - \sin^2 x\right) - 4$$
$$4\sin^2 x + 7\sin x + 3 = 0 \qquad \square$$

[2 marks]

(b)

$$(4\sin x + 3)(\sin x + 1) = 0$$
$$\text{Either } \; 4\sin x + 3 = 0 \; \text{ or } \; \sin x + 1 = 0$$
$$\sin x = -\frac{3}{4} \; \text{ or } \; \sin x = -1$$



$$\text{Solving } \sin x = +\frac{3}{4} \Rightarrow x = \arcsin\left(\frac{3}{4}\right) \Rightarrow x = 48.6° \leftarrow \text{ working angle}$$
$$\therefore \text{ For } \sin x = -\frac{3}{4} \text{ we'll have } x = 228.6°,\; 311.4°$$
$$\text{The other solution } \sin x = -1 \text{ gives } x = 270°$$

$$\text{So, three answers, } x \in \{\, 228.6°,\; 270°,\; 311.4° \,\}$$

[5 marks]