5.5 Answers to 5.4 Exercise

Answer 1

(i)

$$3x^2 + 4x + 2 = 0$$

$$a = 3,\ b = 4 \text{ and } c = 2$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 16 - 4 \times 3 \times 2 \\ &= 16 - 24 \\ &= -8 \end{aligned}$$

[2 marks]

(ii) As $D < 0$, the equation will not have any real solutions

[1 mark]

Answer 2

$$2\tan^2 x - 5\tan x + 4 = 0$$

Let $z = \tan x$ to reveal the underlying quadratic equation as,

$$2z^2 - 5z + 4 = 0$$

$$a = 2,\ b = -5 \text{ and } c = 4$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= 25 - 4 \times 2 \times 4 \\ &= 25 - 32 \\ &= -7 \end{aligned}$$

As $D < 0$, the equation will not have any real solutions □

[3 marks]

Answer 3

(i) The trigonometric identity referred to must be, $cos^2 x + sin^2 x = 1$

$$sin^2 x + 3\,cos\,x - 8 = 0$$

$$\left(1 - cos^2 x\right) + 3\,cos\,x - 8 = 0$$

$$1 - cos^2 x + 3\,cos\,x - 8 = 0$$

$$cos^2 x - 3\,cos\,x + 7 = 0$$

[3 marks]

(ii) $a = 1,\ b = -3$ and $c = 7$

$$D = b^2 - 4ac$$

$$= 9 - 4 \times 1 \times 7$$

$$= 9 - 28$$

$$= -19$$

As $D < 0$, the equation will not have any real solutions □

[3 marks]

Answer 4

(i)

$$3x^2 - 4\sqrt{3}x + 3 = 0$$

$$a = 3,\ b = -4\sqrt{3} \text{ and } c = 3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4\sqrt{3} \pm \sqrt{48 - 4 \times 3 \times 3}}{2 \times 3}$$

$$= \frac{4\sqrt{3} \pm \sqrt{48 - 36}}{6}$$

$$= \frac{4\sqrt{3} \pm \sqrt{12}}{6}$$

$$= \frac{4\sqrt{3} \pm \sqrt{4 \times 3}}{6}$$

$$= \frac{4\sqrt{3} \pm 2\sqrt{3}}{6}$$

$$= \frac{6\sqrt{3}}{6} \quad \text{or} \quad \frac{2\sqrt{3}}{6}$$

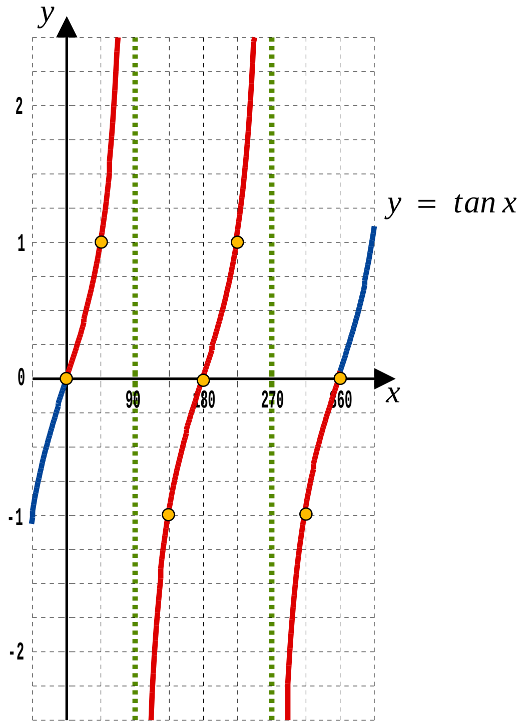
$$= \sqrt{3} \quad \text{or} \quad \frac{\sqrt{3}}{3}$$

[5 marks]

(ii)

$$3\tan^2 x - 4\sqrt{3}\tan x + 3 = 0$$

$$\text{Either} \quad \tan x = \sqrt{3} \quad \text{or} \quad \tan x = \frac{\sqrt{3}}{3}$$



$$\text{For} \quad \tan x = \sqrt{3} \Rightarrow x = 60° \text{ as working angle}$$

$$\text{Then, from the graph, } x = 60°, \; 240°$$

$$\text{For} \quad \tan x = \frac{\sqrt{3}}{3} \Rightarrow x = 30° \text{ as working angle}$$

$$\text{Then, from the graph, } x = 30°, \; 210°$$

In total, four solutions $x \in \{ 30°, 60°, 210°, 240° \}$

[4 marks]

Answer 5

(i)

$$7x^2 + 5\sqrt{7}x + 6 = 0$$

$$a = 7,\ b = 5\sqrt{7} \text{ and } c = 6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5\sqrt{7} \pm \sqrt{175 - 4 \times 7 \times 6}}{2 \times 7}$$

$$= \frac{-5\sqrt{7} \pm \sqrt{175 - 168}}{14}$$

$$= \frac{-5\sqrt{7} \pm \sqrt{7}}{14}$$

$$= -\frac{4\sqrt{7}}{14} \quad \text{or} \quad -\frac{6\sqrt{7}}{14}$$

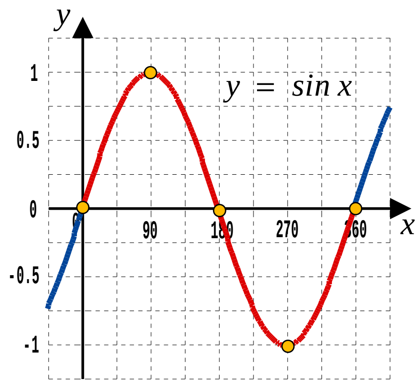
$$= -\frac{2\sqrt{7}}{7} \quad \text{or} \quad -\frac{3\sqrt{7}}{7}$$

[5 marks]

(ii)

$$7\sin^2 x + 5\sqrt{7}\sin x + 6 = 0$$

$$\text{Either } \sin x = -\frac{2\sqrt{7}}{7} \quad \text{or} \quad \sin x = -\frac{3\sqrt{7}}{7}$$



For $\sin x = -\dfrac{3\sqrt{7}}{7}$: No solutions as the $\sin x$ curve is never less than – 1

For $\sin x = +\dfrac{2\sqrt{7}}{7}$: $x = 49.1°$ is the working angle

From the graph this gives two answers: $x \in \{ 229.1°,\ 310.9° \}$

[4 marks]