

7.5 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Trigonometry IV

7.5.1 Solution to 7.2 An Examination Temptation (Teaching Video)

The mistake to avoid is dividing both sides by $\tan x$ because this would be dividing by zero.

In other words, $\tan x = 0$ is a solution to the equation for then $\tan^2 x = 0$ and the LHS = RHS.

Instead, subtract $\tan x$ from both sides and factorise.

$$\tan^2 x = \tan x$$

$$\tan^2 x - \tan x = 0$$

$$\tan x (\tan x - 1) = 0$$

Either $\tan x = 0 \Rightarrow x = 0, 180$: Reject 360 as it's not in the domain

Or $\tan x - 1 = 0 \Rightarrow \tan x = 1 \Rightarrow x = 45, 225$

$$\text{Answers } x \in \{0^\circ, 45^\circ, 180^\circ, 225^\circ\}$$

[5 marks]

7.5.2 Solutions to 7.4 Exercise

Answer 1

$$3 \sin x = 4 \cos x$$

$$\frac{\sin x}{\cos x} = \frac{4}{3}$$

$$\tan x = \frac{4}{3}$$

$$x = \arctan\left(\frac{4}{3}\right)$$

$$x = 53.1^\circ, 233.1^\circ \quad (\text{From a sketch graph of } y = \tan x)$$

[4 marks]

Answer 2

$$\sin x = -4 \cos x$$

$$\frac{\sin x}{\cos x} = -4$$

$$\tan x = -4$$

$$\text{Solve } \tan x = +4$$

$$x = \arctan(+4)$$

$$= 76^\circ \text{ working angle}$$

$$x = 104^\circ, 284^\circ \quad (\text{From a sketch graph of } y = \tan x)$$

[5 marks]

Answer 3

$$5 \sin 2x = 2 \cos 2x$$

$$\frac{\sin 2x}{\cos 2x} = \frac{2}{5}$$

$$\tan 2x = \frac{2}{5}$$

$$2x = \arctan\left(\frac{2}{5}\right)$$

$$2x = 21.8^\circ, 201.8^\circ, 381.8^\circ, 561.8^\circ$$

$$(\text{From a sketch graph of } y = \tan x)$$

$$x = 10.9^\circ, 100.9^\circ, 190.9^\circ, 280.9^\circ$$

[5 marks]

Answer 4

$$5 \sin(3x) - 7 \cos(3x) = 0$$

$$5 \sin(3x) = 7 \cos(3x)$$

$$\frac{\sin(3x)}{\cos(3x)} = \frac{7}{5}$$

$$\tan(3x) = \frac{7}{5}$$

$$3x = \arctan\left(\frac{7}{5}\right)$$

$$3x = 54.5^\circ, 234.5^\circ$$

$$(\text{From a sketch graph of } y = \tan x)$$

$$x = 18.2^\circ, 78.2^\circ$$

[5 marks]

Answer 5**(a)**

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

$$4 (1 - \cos^2 x) + 9 \cos x - 6 = 0$$

$$4 - 4 \cos^2 x + 9 \cos x - 6 = 0$$

$$4 \cos^2 x - 9 \cos x + 2 = 0 \quad \square$$

[2 marks]**(b)**

$$4 \sin^2 x + 9 \cos x - 6 = 0$$

$$4 \cos^2 x - 9 \cos x + 2 = 0 \quad \text{from part (a)}$$

$$(4 \cos x - 1) (\cos x - 2) = 0$$

$$\text{Either } \cos x = \frac{1}{4} \quad \text{or} \quad \cos x = 2$$

$$x = \arccos\left(\frac{1}{4}\right) \quad \cos x = 2 \quad (\text{No solutions})$$

$$x = 75.5^\circ, 284.5^\circ, 435.5^\circ, 644.5^\circ \quad (\text{From sketch graph})$$

[6 marks]**Answer 6**

$$\cos^2 x = \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$$\text{Either } \cos x = 0 \Rightarrow x = 90, 270$$

$$\text{Or } \cos x = 1 \Rightarrow x = 0$$

(360 is not in the domain for the question)

$$\therefore \text{ Answer : } x \in \{ 0, 90, 270 \}$$

[5 marks]**Answer 7**

$$\tan^2 x = 4$$

$$\tan x = \pm 2$$

$$x = \arctan(\pm 2)$$

$$x = 63.4^\circ, 116.6^\circ, 243.4^\circ, 296.6^\circ$$

[5 marks]

Answer 8

Let $z = \tan^2 x$ in which case the given equation becomes,

$$3z^2 - 10z + 3 = 0$$

$$(3z - 1)(z - 3) = 0$$

$$\text{Either } z = \frac{1}{3} \quad \text{or} \quad z = 3$$

$$\tan^2 x = \frac{1}{3} \quad \text{or} \quad \tan^2 x = 3$$

$$\tan x = \pm \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \quad \text{or} \quad \tan x = \pm \sqrt{3}$$

$$= \pm \frac{\sqrt{3}}{3}$$

$$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ \quad x = 60, 120, 240, 300$$

$$x \in \{ 30^\circ, 60^\circ, 120^\circ, 150^\circ, 210^\circ, 240^\circ, 300^\circ, 330^\circ \}$$

[6 marks]