

1.3 Answers to 1.2 Exercise

A-Level Pure Mathematics : Year 2 Integration II

Answer 1

$$(i) \quad \int 4x + \frac{1}{x} dx = 2x^2 + \ln|x| + c$$

[2 marks]

$$(ii) \quad \int \cos(4x) dx = \frac{1}{4} \sin(4x) + c$$

[2 marks]

$$(iii) \quad \int e^{5x} dx = \frac{1}{5} e^{5x} + c$$

[2 marks]

$$(iv) \quad \int \sqrt{x} - \sin(0.5x) dx = \int x^{\frac{1}{2}} - \sin(0.5x) dx \\ = \frac{2x^{\frac{3}{2}}}{3} + 2 \cos(0.5x) + c$$

[4 marks]

$$(v) \quad \int \left(1 + \frac{1}{x}\right)^2 dx = \int 1 + \frac{2}{x} + \frac{1}{x^2} dx \\ = \int 1 + \frac{2}{x} + x^{-2} dx \\ = x + 2 \ln x - x^{-1} + c$$

[4 marks]

Answer 2

$$\begin{aligned} \text{Area} &= \int_1^8 \frac{1}{x} dx \\ &= [\ln|x|]_1^8 \\ &= \ln 8 - \ln 1 \\ &= \ln 2^3 - 0 \\ &= 3 \ln 2 \quad \square \end{aligned}$$

[5 marks]

Answer 3

$$\begin{aligned}\int_0^1 e^{3x} + x^2 dx &= \left[\frac{1}{3} e^{3x} + \frac{x^3}{3} \right]_0^1 \\&= \frac{1}{3} \left[e^{3x} + x^3 \right]_0^1 \\&= \frac{1}{3} \left[e^3 + 1 - e^0 - 0 \right] \\&= \frac{1}{3} (e^3)\end{aligned}$$

[5 marks]

Answer 4

$$\begin{aligned}Area &= \int_0^{\frac{\pi}{4}} 9 \cos(2x) dx \\&= 9 \left[\frac{\sin(2x)}{2} \right]_0^{\frac{\pi}{4}} \\&= \frac{9}{2} \sin\left(\frac{\pi}{2}\right) - 0 \\&= \frac{9}{2}\end{aligned}$$

[5 marks]

Answer 5

(i)

$$\begin{aligned}Area &= \int_0^2 12 e^{6x} dx \\&= 2 \left[e^{6x} \right]_0^2 \\&= 2(e^{12} - e^0) \\&= 2(e^{12} - 1)\end{aligned}$$

□

[5 marks]

(ii)

325 507.583

[1 mark]

Answer 6

$$\begin{aligned} \text{Area} &= \int_0^{\frac{\pi}{6}} \frac{1}{9} \sin\left(\frac{x}{2}\right) dx \\ &= \frac{1}{9} \left[-2 \cos\left(\frac{x}{2}\right) \right]_0^{\frac{\pi}{6}} \\ &= \frac{1}{9} \left\{ \left(-2 \cos\left(\frac{\pi}{12}\right) \right) - \left(-2 \cos 0 \right) \right\} \\ &= 0.00757 \quad (3 \text{ significant figures}) \end{aligned}$$

[5 marks]