

2.3 Answers

A-Level Pure Mathematics : Year 2

Integration II

2.3.1 Solutions to 2.1 Examples (Teaching Video)

Example N° 2

Add in a fiddle factor of $\frac{1}{3} \times 3$

$$\begin{aligned}\int (3x + 1)^4 dx &= \frac{1}{3} \int 3 (3x + 1)^4 dx \\&= \frac{1}{3} \times \frac{(3x + 1)^5}{5} + c \\&= \frac{(3x + 1)^5}{15} + c\end{aligned}$$

[3 marks]

Example N° 3

Add in a fiddle factor of $\frac{1}{4} \times 4$

$$\begin{aligned}\int \frac{24}{(4x - 2)^4} dx &= 24 \int (4x - 2)^{-4} dx \\&= 24 \times \frac{1}{4} \int 4 (4x - 2)^{-4} dx \\&= 6 \times \frac{(4x - 2)^{-3}}{(-3)} + c \\&= -2 (4x - 2)^{-3} + c \\&= -\frac{2}{(4x - 2)^3} + c\end{aligned}$$

[3 marks]

2.3.2 Solutions to 3.2 Exercise

Answer 1

Add in a fiddle factor of $\frac{1}{3} \times 3$

$$\begin{aligned}\int (3x + 2)^5 dx &= \frac{1}{3} \int 3 (3x + 2)^5 dx \\&= \frac{1}{3} \times \frac{(3x + 2)^6}{6} + c \\&= \frac{(3x + 2)^6}{18} + c\end{aligned}$$

[3 marks]

Answer 2

Add in a fiddle factor of $\frac{1}{7} \times 7$

$$\begin{aligned}\int (7x - 3)^3 dx &= \frac{1}{7} \int 7 (7x - 3)^3 dx \\ &= \frac{1}{7} \times \frac{(7x - 3)^4}{4} + c \\ &= \frac{(7x - 3)^4}{28} + c\end{aligned}$$

[3 marks]

Answer 3

Add in a fiddle factor of $\frac{1}{5} \times 5$

$$\begin{aligned}60 \int (4 + 5x)^3 dx &= 60 \times \frac{1}{5} \int 5 (4 + 5x)^3 dx \\ &= 12 \times \frac{(4 + 5x)^4}{4} + c \\ &= 3 (4 + 5x)^4 + c\end{aligned}$$

[3 marks]

Answer 4

Add in a fiddle factor of $-\frac{1}{2} \times -2$

$$\begin{aligned}\int (3 - 2x)^5 dx &= -\frac{1}{2} \int (-2) (3 - 2x)^5 dx \\ &= -\frac{1}{2} \times \frac{(3 - 2x)^6}{6} + c \\ &= -\frac{(3 - 2x)^6}{12} + c\end{aligned}$$

[3 marks]

Answer 5

Add in a fiddle factor of $-2 \times -\frac{1}{2}$

$$\begin{aligned}\int \left(7 - \frac{1}{2}x\right)^6 dx &= (-2) \int \left(-\frac{1}{2}\right) \left(7 - \frac{1}{2}x\right)^6 dx \\ &= (-2) \frac{\left(7 - \frac{1}{2}x\right)^7}{7} + c \\ &= -\frac{2}{7} \left(7 - \frac{1}{2}x\right)^7 + c\end{aligned}$$

[3 marks]

Answer 6

Add in a fiddle factor of $\frac{1}{4} \times 4$

$$\begin{aligned}
 18 \int \sqrt{1 + 4x} \, dx &= 18 \times \frac{1}{4} \int 4 (1 + 4x)^{\frac{1}{2}} \, dx \\
 &= \frac{9}{2} \times \frac{2 (1 + 4x)^{\frac{3}{2}}}{3} + c \\
 &= 3 (1 + 4x)^{\frac{3}{2}} + c
 \end{aligned}$$

[3 marks]

Answer 7

Add in a fiddle factor of $\frac{3}{2} \times \frac{2}{3}$

$$\begin{aligned}
 \int \left(\frac{2x}{3} + 8 \right)^{\frac{1}{2}} \, dx &= \frac{3}{2} \int \frac{2}{3} \left(\frac{2x}{3} + 8 \right)^{\frac{1}{2}} \, dx \\
 &= \frac{3}{2} \times \frac{2 \left(\frac{2x}{3} + 8 \right)^{\frac{3}{2}}}{3} + c \\
 &= \left(\frac{2x}{3} + 8 \right)^{\frac{3}{2}} + c
 \end{aligned}$$

[3 marks]

Answer 8

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}
 \int \frac{1}{(2x - 1)^4} \, dx &= \int (2x - 1)^{-4} \, dx \\
 &= \frac{1}{2} \int 2 (2x - 1)^{-4} \, dx \\
 &= \frac{1}{2} \times \frac{(2x - 1)^{-3}}{(-3)} + c \\
 &= -\frac{1}{6 (2x - 1)^3} + c
 \end{aligned}$$

[3 marks]

Answer 9

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}
 \int 6 (2x - 1)^{-2} dx &= 6 \int (2x - 1)^{-2} dx \\
 &= 6 \times \frac{1}{2} \int 2 (2x - 1)^{-2} dx \\
 &= 3 \int 2 (2x - 1)^{-2} dx \\
 &= 3 \times \frac{(2x - 1)^{-1}}{(-1)} + c \\
 &= -\frac{3}{(2x - 1)} + c
 \end{aligned}$$

[3 marks]

Answer 10

Add in a fiddle factor of $\frac{1}{3} \times 3$

$$\begin{aligned}
 \int \frac{6}{\sqrt{3x - 1}} dx &= 6 \int (3x - 1)^{-\frac{1}{2}} dx \\
 &= 6 \times \frac{1}{3} \int 3 (3x - 1)^{-\frac{1}{2}} dx \\
 &= 2 \int 3 (3x - 1)^{-\frac{1}{2}} dx \\
 &= 2 \times 2 (3x - 1)^{\frac{1}{2}} + c \\
 &= 4\sqrt{3x - 1} + c
 \end{aligned}$$

[3 marks]

Answer 11

No fiddle factor needed !

$$\begin{aligned}
 \int \frac{1}{4 (x + 3)^2} dx &= \frac{1}{4} \int (x + 3)^{-2} dx \\
 &= \frac{1}{4} \times \frac{(x + 3)^{-1}}{(-1)} + c \\
 &= -\frac{1}{4 (x + 3)} + c
 \end{aligned}$$

[3 marks]

Answer 12

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}
 \int 14 (3 + 2x)^{-2} dx &= 14 \int (3 + 2x)^{-2} dx \\
 &= 14 \times \frac{1}{2} \int 2 (3 + 2x)^{-2} dx \\
 &= 7 \int (3 + 2x)^{-2} dx \\
 &= 7 \times \frac{(3 + 2x)^{-1}}{(-1)} + c \\
 &= -\frac{7}{(3 + 2x)} + c
 \end{aligned}$$

[3 marks]

Answer 13

Add in a fiddle factor of $-\frac{1}{2} \times -2$

$$\begin{aligned}
 \int \frac{1}{(1 - 2x)^{\frac{3}{2}}} dx &= -\frac{1}{2} \int (-2) (1 - 2x)^{-\frac{3}{2}} dx \\
 &= -\frac{1}{2} \times \frac{2 (1 - 2x)^{-\frac{1}{2}}}{(-1)} + c \\
 &= \frac{1}{\sqrt{1 - 2x}} + c
 \end{aligned}$$

[3 marks]

Answer 14

Add in a fiddle factor of $\frac{1}{4} \times 4$

$$\begin{aligned}
 \int_0^1 (4x + 1)^4 dx &= \frac{1}{4} \int_0^1 4 (4x + 1)^4 dx \\
 &= \frac{1}{4} \left[\frac{(4x + 1)^5}{5} \right]_0^1 \\
 &= \frac{1}{4} \left\{ [625] - \left[\frac{1}{5} \right] \right\} \\
 &= 156.2
 \end{aligned}$$

[3 marks]

Answer 15

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}
 \int_0^4 \sqrt{2x+1} \, dx &= \frac{1}{2} \int_0^4 2 (2x+1)^{\frac{1}{2}} \, dx \\
 &= \frac{1}{2} \left[\frac{2 (2x+1)^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\
 &= \frac{1}{3} \left[(2x+1)^{\frac{3}{2}} \right]_0^4 \\
 &= \frac{1}{3} \{ [27] - [1] \} \\
 &= \frac{26}{3} \quad \text{or} \quad 8\frac{2}{3}
 \end{aligned}$$

[4 marks]

Answer 16

No fiddle factor needed !

$$\begin{aligned}
 \int_2^5 \frac{1}{(x-1)^3} \, dx &= \int_2^5 (x-1)^{-3} \, dx \\
 &= \left[\frac{(x-1)^{-2}}{(-2)} \right]_2^5 \\
 &= -\frac{1}{2} \left[\frac{1}{(x-1)^2} \right]_2^5 \\
 &= -\frac{1}{2} \left\{ \left[\frac{1}{16} \right] - \left[\frac{1}{1} \right] \right\} \\
 &= -\frac{1}{2} \times \left\{ -\frac{15}{16} \right\} \\
 &= \frac{15}{32}
 \end{aligned}$$

[4 marks]