

3.3 Answers

A-Level Pure Mathematics : Year 2 Integration II

3.3.1 Solutions to 3.1 Examples

Example N° 1

Add in a fiddle factor of $\frac{1}{12} \times 12$

$$\begin{aligned}\int 120 x^3 (3x^4 + 7)^4 dx &= 120 \int x^3 (3x^4 + 7)^4 dx \\&= 120 \times \frac{1}{12} \int 12 x^3 (3x^4 + 7)^4 dx \\&= 10 \times \frac{(3x^4 + 7)^5}{5} + c \\&= 2(3x^4 + 7)^5 + c\end{aligned}$$

[3 marks]

Example N° 2

Add in a fiddle factor of $\frac{1}{4} \times 4$

$$\begin{aligned}\int 64 (x^3 - 2)(x^4 - 8x)^3 dx &= 64 \int (x^3 - 2)(x^4 - 8x)^3 dx \\&= 64 \times \frac{1}{4} \int 4 (x^3 - 2)(x^4 - 8x)^3 dx \\&= 16 \int (4x^3 - 8)(x^4 - 8x)^3 dx \\&= 16 \times \frac{(x^4 - 8x)^4}{4} + c \\&= 4(x^4 - 8x)^4 + c\end{aligned}$$

[4 marks]

3.3.2 Solutions to 3.2 Exercise

Answer 1

Add in a fiddle factor of $\frac{1}{6} \times 6$

$$\begin{aligned}\int 150x(3x^2 + 1)^4 dx &= 150 \int x(3x^2 + 1)^4 dx \\&= 150 \times \frac{1}{6} \int 6x(3x^2 + 1)^4 dx \\&= 25 \times \frac{(3x^2 + 1)^5}{5} + c \\&= 5(3x^2 + 1)^5 + c\end{aligned}$$

[3 marks]

Answer 2

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}\int 56x(x^2 - 5)^3 dx &= 56 \int x(x^2 - 5)^3 dx \\&= 56 \times \frac{1}{2} \int 2x(x^2 - 5)^3 dx \\&= 28 \times \frac{(x^2 - 5)^4}{4} + c \\&= 7(x^2 - 5)^4 + c\end{aligned}$$

[3 marks]

Answer 3

Add in a fiddle factor of $\left(-\frac{1}{3}\right) \times (-3)$

$$\begin{aligned}\int 42x^2(2 - x^3)^6 dx &= 42 \int x^2(2 - x^3)^6 dx \\&= 42 \times \left(-\frac{1}{3}\right) \int (-3)x^2(2 - x^3)^6 dx \\&= -14 \times \frac{(2 - x^3)^7}{7} + c \\&= -2(2 - x^3)^7 + c\end{aligned}$$

[3 marks]

Answer 4

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}
 \int 12x \sqrt{x^2 + 6} \, dx &= 12 \int x(x^2 + 6)^{\frac{1}{2}} \, dx \\
 &= 12 \times \frac{1}{2} \int 2x(x^2 + 6)^{\frac{1}{2}} \, dx \\
 &= 6 \times \frac{2(x^2 + 6)^{\frac{3}{2}}}{\frac{3}{2}} + c \\
 &= 4(x^2 + 6)^{\frac{3}{2}} + c
 \end{aligned}$$

[3 marks]

Answer 5

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}
 \int \frac{21x}{(1 + x^2)^2} \, dx &= 21 \int x(1 + x^2)^{-2} \, dx \\
 &= 21 \times \frac{1}{2} \int 2x(1 + x^2)^{-2} \, dx \\
 &= \frac{21}{2} \times \frac{(1 + x^2)^{-1}}{(-1)} + c \\
 &= -\frac{21}{2(1 + x^2)} + c
 \end{aligned}$$

[3 marks]

Answer 6

Add in a fiddle factor of $\frac{1}{4} \times 4$

$$\begin{aligned}
 \int \frac{12x^3}{\sqrt{3 + x^4}} \, dx &= \int 12x^3(3 + x^4)^{-\frac{1}{2}} \, dx \\
 &= 12 \int x^3(3 + x^4)^{-\frac{1}{2}} \, dx \\
 &= 12 \times \frac{1}{4} \int 4x^3(3 + x^4)^{-\frac{1}{2}} \, dx \\
 &= 3 \times \frac{2(3 + x^4)^{\frac{1}{2}}}{\frac{1}{2}} + c \\
 &= 6\sqrt{3 + x^4} + c
 \end{aligned}$$

[3 marks]

Answer 7

Add in a fiddle factor of $\frac{1}{3} \times 3$

$$\begin{aligned}
 \int 60(x^2 - 1)(x^3 - 3x + 7)^4 dx &= 60 \int (x^2 - 1)(x^3 - 3x + 7)^4 dx \\
 &= 60 \times \frac{1}{3} \int 3(x^2 - 1)(x^3 - 3x + 7)^4 dx \\
 &= 20 \int (3x^2 - 3)(x^3 - 3x + 7)^4 dx \\
 &= 20 \times \frac{(x^3 - 3x + 7)^5}{5} + c \\
 &= 4(x^3 - 3x + 7)^5 + c
 \end{aligned}$$

[4 marks]

Answer 8

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}
 \int \frac{x + 1}{(x^2 + 2x + 3)^4} dx &= \int (x + 1)(x^2 + 2x + 3)^{-4} dx \\
 &= \frac{1}{2} \int 2(x + 1)(x^2 + 2x + 3)^{-4} dx \\
 &= \frac{1}{2} \int (2x + 2)(x^2 + 2x + 3)^{-4} dx \\
 &= \frac{1}{2} \frac{(x^2 + 2x + 3)^{-3}}{(-3)} + c \\
 &= -\frac{1}{6(x^2 + 2x + 3)^3} + c
 \end{aligned}$$

[4 marks]

Answer 9

Add in a fiddle factor of $\frac{1}{3} \times 3$

$$\begin{aligned}
 \int \frac{x^2 + 2}{\sqrt{x^3 + 6x}} dx &= \int (x^2 + 2)(x^3 + 6x)^{-\frac{1}{2}} dx \\
 &= \frac{1}{3} \int 3(x^2 + 2)(x^3 + 6x)^{-\frac{1}{2}} dx \\
 &= \frac{1}{3} \int (3x^2 + 6)(x^3 + 6x)^{-\frac{1}{2}} dx \\
 &= \frac{1}{3} \times \frac{2(x^3 + 6x)^{\frac{1}{2}}}{1} + c \\
 &= \frac{2}{3} \sqrt{x^3 + 6x} + c
 \end{aligned}$$

[4 marks]

Answer 10

$$(i) \quad \frac{dy}{dx} = (3x^2 + 9) \cos(x^3 + 9x)$$

[3 marks]

$$\begin{aligned}
 (ii) \quad \int (x^2 + 3) \cos(x^3 + 9x) dx &= \frac{1}{3} \int 3(x^2 + 3) \cos(x^3 + 9x) dx \\
 &= \frac{1}{3} \int (3x^2 + 9) \cos(x^3 + 9x) dx \\
 &= \frac{1}{3} \sin(x^3 + 9x) + c \quad \text{From part (i)}
 \end{aligned}$$

[3 marks]

$$\begin{aligned}
 (iii) \quad \int x^2 \cos(4x^3 - 7) dx &= \frac{1}{12} \int 12x^2 \cos(4x^3 - 7) dx \\
 &= \frac{1}{12} \sin(4x^3 - 7) + c
 \end{aligned}$$

[4 marks]