

4.3 Answers

A-Level Pure Mathematics : Year 2

Integration II

4.3.1 Solution to 4.1 Example (Teaching Video)

Add in a fiddle factor of $\frac{1}{12} \times 12$

$$\begin{aligned}\int \frac{36x^2}{4x^3 - 9} dx &= 36 \int x^2 (4x^3 - 9)^{-1} dx \\ &= 36 \times \frac{1}{12} \int 12x^2 (4x^3 - 9)^{-1} dx \\ &= 3 \int 12x^2 (4x^3 - 9)^{-1} dx \\ &= 3 \ln |4x^3 - 9| + c\end{aligned}$$

[3 marks]

4.3.2 Solutions to 4.2 Exercise

Answer 1

Add in a fiddle factor of $\frac{1}{8} \times 8$

$$\begin{aligned}\int \frac{40x^3}{1 + 2x^4} dx &= 40 \int x^3 (1 + 2x^4)^{-1} dx \\ &= 40 \times \frac{1}{8} \int 8x^3 (1 + 2x^4)^{-1} dx \\ &= 5 \ln |1 + 2x^4| + c\end{aligned}$$

[3 marks]

Answer 2

Add in a fiddle factor of $\left(-\frac{1}{5}\right) \times (-5)$

$$\begin{aligned}\int \frac{35x^4}{3 - x^5} dx &= 35 \int x^4 (3 - x^5)^{-1} dx \\ &= 35 \times \left(-\frac{1}{5}\right) \int (-5) x^4 (3 - x^5)^{-1} dx \\ &= -7 \ln |3 - x^5| + c\end{aligned}$$

[3 marks]

Answer 3

Add in a fiddle factor of $\frac{1}{4} \times 4$

$$\begin{aligned}
 \int \frac{x^3 + 1}{x^4 + 4x} dx &= \int (x^3 + 1)(x^4 + 4x)^{-1} dx \\
 &= \frac{1}{4} \int 4(x^3 + 1)(x^4 + 4x)^{-1} dx \\
 &= \frac{1}{4} \int (4x^3 + 4)(x^4 + 4x)^{-1} dx \\
 &= \frac{1}{4} \ln|x^4 + 4x| + c
 \end{aligned}$$

[4 marks]

Answer 4

Add in a fiddle factor of $\frac{1}{4} \times 4$

$$\begin{aligned}
 \int \frac{8x^3}{(x^2 + 1)(x^2 - 1)} dx &= 8 \int \frac{x^3}{x^4 - 1} dx \\
 &= 8 \times \frac{1}{4} \int 4x^3 (x^4 - 1)^{-1} dx \\
 &= 2 \int 4x^3 (x^4 - 1)^{-1} dx \\
 &= 2 \ln|x^4 - 1| + c
 \end{aligned}$$

[4 marks]

Answer 5

(i) $\int \frac{x + 3}{x^2 + x} dx = \int (x + 3)(x^2 + x)^{-1} dx$

- If $f(x) = x^2 + x$ then $f'(x) = 2x + 1$
- But we have $x + 3$ where we want $2x + 1$
- The $x + 3$ isn't adjustable into $2x + 1$ by a fiddle factor

[2 marks]

(ii)

$$\begin{aligned}
 \text{RHS} &= \frac{3}{x} - \frac{2}{x + 1} \\
 &= \frac{3}{x} \times \frac{x + 1}{x + 1} - \frac{2}{x + 1} \times \frac{x}{x} \\
 &= \frac{3x + 3 - 2x}{x(x + 1)} \\
 &= \frac{x + 3}{x(x + 1)} \\
 &= \text{LHS} \quad \square
 \end{aligned}$$

[2 marks]

(iii)

$$\begin{aligned}\int \frac{x+3}{x^2+x} dx &= \int \frac{3}{x} dx - \int \frac{2}{x+1} dx \\&= 3 \int \frac{1}{x} dx - 2 \int \frac{1}{x+1} dx \\&= 3 \ln |x| - 2 \ln |x+1| + c \\&= \ln |x|^3 - \ln |x+1|^2 + c \\&= \ln \left| \frac{x^3}{(x+1)^2} \right| + c\end{aligned}$$

[4 marks]

Answer 6

Add in a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}\int_1^3 \frac{6x^3 + 5x}{3x^4 + 5x^2 + 1} dx &= \int_1^3 (6x^3 + 5x)(3x^4 + 5x^2 + 1)^{-1} dx \\&= \frac{1}{2} \int_1^3 2(6x^3 + 5x)(3x^4 + 5x^2 + 1)^{-1} dx \\&= \frac{1}{2} \int_1^3 (12x^3 + 10x)(3x^4 + 5x^2 + 1)^{-1} dx \\&= \frac{1}{2} \left[\ln |3x^4 + 5x^2 + 1| \right]_1^3 \\&= \frac{1}{2} [\ln |289| - \ln |9|] \\&= \frac{1}{2} \ln \left(\frac{289}{9} \right) \\&= \ln \left(\frac{289}{9} \right)^{\frac{1}{2}} \\&= \ln \left(\frac{17}{3} \right) \quad \square\end{aligned}$$

[8 marks]