6.3 Answers

6.3.1 Solution to 6.1 Example

Apply a fiddle factor of $\frac{1}{21} \times 21$

$$\begin{aligned}\int 63x^2 e^{7x^3}\,dx &= 63\int x^2 e^{7x^3}\,dx \\ &= 63 \times \frac{1}{21}\int 21x^2 e^{7x^3}\,dx \\ &= 3\int 21x^2 e^{7x^3}\,dx \\ &= 3e^{7x^3} + c\end{aligned}$$

[3 marks]

6.3.2 Solution to 6.2 Exercise

Answer 1

Apply a fiddle factor of $\frac{1}{10} \times 10$

$$\begin{aligned}\int 30x\, e^{5x^2}\,dx &= 30\int x\, e^{5x^2}\,dx \\ &= 30 \times \frac{1}{10}\int 10x\, e^{5x^2}\,dx \\ &= 3e^{5x^2} + c\end{aligned}$$

[3 marks]

Answer 2

Apply a fiddle factor of $\frac{1}{3} \times 3$

$$\begin{aligned}\int_0^2 x^2 e^{x^3}\,dx &= \frac{1}{3}\int_0^2 3x^2 e^{x^3}\,dx \\ &= \frac{1}{3}\left[\, e^{x^3} \right]_0^2 \\ &= \frac{1}{3}\left\{ e^8 - 1 \right\} \\ &= 993 \text{ to the nearest integer}\end{aligned}$$

[4 marks]

Answer 3

Apply a fiddle factor of $\frac{1}{20} \times 20$

$$\begin{aligned}\int 100\,x^3 \cos\left(6 + 5x^4\right)\,dx &= 100 \int x^3 \cos\left(6 + 5x^4\right)\,dx \\ &= 100 \times \frac{1}{20} \int 20\,x^3 \cos\left(6 + 5x^4\right)\,dx \\ &= 5 \int 20\,x^3 \cos\left(6 + 5x^4\right)\,dx \\ &= 5 \sin(6 + 5x^4) + c\end{aligned}$$

[4 marks]

Answer 4

Apply a fiddle factor of $\frac{1}{2} \times 2$

$$\begin{aligned}\int \frac{3}{2}\,x \sin\left(x^2\right)\,dx &= \frac{3}{2} \times \frac{1}{2} \int 2x \sin\left(x^2\right)\,dx \\ &= \frac{3}{4} \int 2x \sin\left(x^2\right)\,dx \\ &= \frac{3}{4}\left(-\cos\left(x^2\right)\right) + c \\ &= -\frac{3}{4} \cos\left(x^2\right) + c\end{aligned}$$

[4 marks]

Answer 5

Apply a fiddle factor of $2 \times \frac{1}{2}$

$$\begin{aligned}\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= \int x^{-\frac{1}{2}} \cos\left(x^{\frac{1}{2}}\right)\,dx \\ &= 2 \int \frac{1}{2}\,x^{-\frac{1}{2}} \cos\left(x^{\frac{1}{2}}\right)\,dx \\ &= 2 \sin\left(x^{\frac{1}{2}}\right) + c\end{aligned}$$

[4 marks]

Answer 6

Apply a fiddle factor of $\frac{1}{3} \times 3$

$$\int_1^5 \frac{12}{\sqrt{3x+1}}\,dx = 12\int_1^5 (3x+1)^{-\frac{1}{2}}\,dx$$

$$= 12 \times \frac{1}{3}\int_1^5 3(3x+1)^{-\frac{1}{2}}\,dx$$

$$= 4\left[\frac{2(3x+1)^{\frac{1}{2}}}{1}\right]_1^5$$

$$= 8\left[(3x+1)^{\frac{1}{2}}\right]_1^5$$

$$= 8\{4-2\}$$

$$= 16$$

[7 marks]

Answer 7

Apply a fiddle factor of $2 \times \frac{1}{2}$

$$\int_0^{2\sqrt{\pi}} x\sin\left(\frac{x^2}{4}\right)dx = 2\int_0^{2\sqrt{\pi}} \frac{1}{2}x\sin\left(\frac{x^2}{4}\right)dx$$

$$= 2\left[-\cos\left(\frac{x^2}{4}\right)\right]_0^{2\sqrt{\pi}}$$

$$= (-2)\{[\cos\pi] - [\cos 0]\}$$

$$= (-2)\{[(-1)] - [1]\}$$

$$= (-2)\{-2\}$$

$$= 4$$

[7 marks]

Answer 8
(i)

$$y = e^{\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} e^{\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

[2 marks]

(ii)

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2\int_1^4 \frac{e^{\sqrt{x}}}{2\sqrt{x}}\, dx$$

$$= 2\left[e^{\sqrt{x}} \right]_1^4$$

$$= 2\left\{ e^2 - e^1 \right\}$$

$$= 9.342 \qquad \text{to 3 decimal places}$$

[5 marks]