Grade Gobbler 1

Answer 1

$$\int_2^3 x^3 + 7 \; dx = \left[\frac{x^4}{4} + 7x \right]_2^3$$

$$= \left(\left[\frac{81}{4} + 7 \times 3 \right] - \left[4 + 7 \times 2 \right] \right)$$

$$= \frac{93}{4} \text{ or } 23.25$$

You will get NO MARKS in this question if you use the "integration" button on a calculator to get this numerical solution and have no working.

[4 marks]

Answer 2

(i) 16.25

[1 mark]

(ii) $f(x) = x^4 + x^{-2}$

$$\therefore \; f'(x) = 4x^3 - 2x^{-3}$$

$$= 4x^3 - \frac{2}{x^3}$$

[2 marks]

(iii) 31.75

[1 mark]

Answer 3

(i)

$$(2 + 3x)^4 = (1) \times (2)^4 \times (3x)^0$$
$$+ (4) \times (2)^3 \times (3x)^1$$
$$+ (6) \times (2)^2 \times (3x)^2$$
$$+ (4) \times (2)^1 \times (3x)^3$$
$$+ (1) \times (2)^0 \times (3x)^4$$
$$= 16 + 96x + 216x^2 + 216x^3 + 81x^4$$

[4 marks]

(ii) $\left(2 + 3\sqrt{x}\right)^4 = 16 + 96\sqrt{x} + 216x + 216\left(\sqrt{x}\right)^3 + 81x^2$

[2 marks]

Answer 4

(i)

$$f(3) = 3^3 + 7 \times 3^2 - 6 \times 3 - 72$$
$$= 27 + 63 - 18 - 72$$
$$= 0 \quad \therefore (x - 3) \text{ is a factor, by the factor theorem} \; \square$$

[2 marks]

(ii) $x^3 + 7x^2 - 6x - 72 = (x - 3)(x^2 + 10x + 24)$

$= (x - 3)(x + 4)(x + 6)$

[3 marks]

Answer 5

$$v = u + at$$
$$19 = 2 + a \times 5$$
$$17 = 5a$$
$$a = 3.4 \text{ m s}^{-2}$$

[4 marks]

Answer 6

$$\int \sqrt{x} + 4(x + 2)(x - 3)\,dx = \int x^{\frac{1}{2}} + 4(x^2 - x - 6)\,dx$$

$$= \int x^{\frac{1}{2}} + 4x^2 - 4x - 24\,dx$$

$$= \frac{2x^{\frac{3}{2}}}{3} + \frac{4x^3}{3} - 2x^2 - 24x + c$$

[5 marks]

Answer 7

(i) $f(x) = x^3 - 3x + 1$

$\Rightarrow \quad f'(x) = 3x^2 - 3$

We require $3x^2 - 3 = 0$

$x^2 = 1$ so $x = \pm 1$

Turning points are $(-1, +3), (1, -1)$

[5 marks]

(ii) $f''(x) = 6x$ is the "bend detector" equation

$(-1, +3)$ is a local maximum as $f''(-1)$ is negative

$(1, -1)$ is a local minimum as $f''(1)$ is positive

[3 marks]

Answer 8

The triangle will be right angled as Pythagoras' theorem holds.

i.e. $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$

which is proven by expanding all brackets to show LHS = RHS.

This relationship gives a method of finding Pythagorean Integer Triples.

[4 marks]