

Grade Gobbler 1

Answer 1

$$\begin{aligned}\int_2^3 x^3 + 7 \, dx &= \left[\frac{x^4}{4} + 7x \right]_2^3 \\&= \left(\left[\frac{81}{4} + 7 \times 3 \right] - [4 + 7 \times 2] \right) \\&= \frac{93}{4} \text{ or } 23.25\end{aligned}$$

You will get NO MARKS in this question if you use the “integration” button on a calculator to get this numerical solution and have no working.

[4 marks]

Answer 2

(i) 16.25

[1 mark]

(ii) $f(x) = x^4 + x^{-2}$
 $\therefore f'(x) = 4x^3 - 2x^{-3}$
 $= 4x^3 - \frac{2}{x^3}$

[2 marks]

(iii) 31.75

[1 mark]

Answer 3

(i)
$$\begin{aligned}(2 + 3x)^4 &= (1) \times (2)^4 \times (3x)^0 \\&\quad + (4) \times (2)^3 \times (3x)^1 \\&\quad + (6) \times (2)^2 \times (3x)^2 \\&\quad + (4) \times (2)^1 \times (3x)^3 \\&\quad + (1) \times (2)^0 \times (3x)^4 \\&= 16 + 96x + 216x^2 + 216x^3 + 81x^4\end{aligned}$$

[4 marks]

(ii) $(2 + 3\sqrt{x})^4 = 16 + 96\sqrt{x} + 216x + 216(\sqrt{x})^3 + 81x^2$

[2 marks]

Answer 4

(i)
$$\begin{aligned}f(3) &= 3^3 + 7 \times 3^2 - 6 \times 3 - 72 \\&= 27 + 63 - 18 - 72 \\&= 0 \quad \therefore (x - 3) \text{ is a factor, by the factor theorem} \quad \square\end{aligned}$$

[2 marks]

$$\begin{aligned}
 \text{(ii)} \quad x^3 + 7x^2 - 6x - 72 &= (x - 3)(x^2 + 10x + 24) \\
 &= (x - 3)(x + 4)(x + 6)
 \end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned}
 v &= u + at \\
 19 &= 2 + a \times 5 \\
 17 &= 5a \\
 a &= 3.4 \text{ m s}^{-2}
 \end{aligned}$$

[4 marks]

Answer 6

$$\begin{aligned}
 \int \sqrt{x} + 4(x + 2)(x - 3) dx &= \int x^{\frac{1}{2}} + 4(x^2 - x - 6) dx \\
 &= \int x^{\frac{1}{2}} + 4x^2 - 4x - 24 dx \\
 &= \frac{2x^{\frac{3}{2}}}{3} + \frac{4x^3}{3} - 2x^2 - 24x + c
 \end{aligned}$$

[5 marks]

Answer 7

$$\begin{aligned}
 \text{(i)} \quad f(x) &= x^3 - 3x + 1 \\
 \Rightarrow f'(x) &= 3x^2 - 3
 \end{aligned}$$

$$\text{We require } 3x^2 - 3 = 0$$

$$x^2 = 1 \text{ so } x = \pm 1$$

Turning points are $(-1, +3), (1, -1)$

[5 marks]

$$\begin{aligned}
 \text{(ii)} \quad f''(x) &= 6x \text{ is the "bend detector" equation} \\
 (-1, +3) &\text{ is a local maximum as } f''(-1) \text{ is negative} \\
 (1, -1) &\text{ is a local minimum as } f''(1) \text{ is positive}
 \end{aligned}$$

[3 marks]

Answer 8

The triangle will be right angled as Pythagoras' theorem holds.

$$\text{i.e. } (m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$$

which is proven by expanding all brackets to show LHS = RHS.

This relationship gives a method of finding Pythagorean Integer Triples.

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk