Answers to Grade Gobbler 4

Answer 1
(i)

$$m = \frac{\Delta y}{\Delta x} = \frac{2 - 5}{8 - 2} = \frac{-3}{6} = -\frac{1}{2}$$

$$y = mx + c \Rightarrow 5 = -\frac{1}{2} \times 2 + c \qquad \therefore c = 6$$

$$y = -\frac{1}{2} x + 6$$

[2 marks]

(ii)

Perpendicular gradient will be $+2$

$$y = 2x + 4$$

[2 marks]

Answer 2

$$y = \int 4 + x - 3x^2 \; dx$$

$$y = 4x + \frac{x^2}{2} - x^3 + c$$

Told that (2, 5) is on this curve...

$$5 = 4 \times 2 + \frac{2^2}{2} - 2^3 + c$$

$$\therefore c = 3$$

And the equation of the curve is thus;

$$y = 4x + \frac{x^2}{2} - x^3 + 3$$

[4 marks]

Answer 3

$$\left| AC \right| = \sqrt{4^2 + 5^2}$$

$$= \sqrt{41}$$

$$\angle VAC = arctan\left(\frac{8}{\frac{1}{2}\sqrt{41}} \right)$$

$$= 68.2°$$

[4 marks]

Answer 4

$$\frac{1}{(1+\sqrt{x})} \times \frac{(1-\sqrt{x})}{(1-\sqrt{x})} + \frac{1}{(1-\sqrt{x})} \times \frac{(1+\sqrt{x})}{(1+\sqrt{x})}$$

$$= \frac{(1-\sqrt{x}) + (1+\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})}$$

$$= \frac{2}{1-x}$$

[4 marks]

Answer 5

(i) $f(x) = x^3 - 5x^2 + 3x + 9$

The 9 on the end suggests the possible factors are ($x \pm 1$), ($x \pm 3$) and ($x \pm 9$)
We're looking for a value of x that causes $f(x)$ to equal zero.

$f(1) = 8$ which is no good but $f(-1) = 0$ and so $(x+1)$ is a factor

Thus $x^3 - 5x^2 + 3x + 9 = (x+1)(x^2 - 6x + 9)$

$$= (x+1)(x-3)^2$$

[5 marks]

(ii) $x = -1,\ x = 3$ are the solutions

You will get NO MARKS in this question if you use the “solve equation” facility on a calculator to get these solutions without any working.

[2 marks]

Answer 6

$$\cos 2\theta = 0.413$$

$$2\theta = \arccos 0.413$$

$$2\theta = 65.6°,\quad 294.4°,\quad 425.6°,\quad 654.4°$$

$$\theta = 32.8°,\quad 147.2°,\quad 212.8°,\quad 327.2°$$

[5 marks]

Answer 7

$$\left(x + \frac{5}{x}\right)^3 = (1) \times (x)^3 \times \left(\frac{5}{x}\right)^0$$

$$+ (3) \times (x)^2 \times \left(\frac{5}{x}\right)^1$$

$$+ (3) \times (x)^1 \times \left(\frac{5}{x}\right)^2$$

$$+ (1) \times (x)^0 \times \left(\frac{5}{x}\right)^3$$

$$= x^3 + 15x + \frac{75}{x} + \frac{125}{x^3}$$

[4 marks]

Answer 8
(i)

$$y = x^3 - x$$

$$\text{When } x = 2, \quad y = 2^3 - 2$$
$$= 8 - 2$$
$$= 6$$

$$\therefore \ (2, \ 6) \text{ is on the curve} \qquad \square$$

[1 mark]

(ii)

$$\frac{dy}{dx} = 3x^2 - 1$$

$$\text{When } x = 2, \quad \frac{dy}{dx} = 11$$

$$y = mx + c \Rightarrow 6 = 11 \times 2 + c$$

$$\therefore c = -16$$

$$\text{The equation of the tangent is } \ y = 11x - 16$$

[3 marks]

(iii)

$$\text{Key step :} \qquad x^3 - x = 11x - 16$$

$$x^3 - 12x + 16 = 0$$

The 16 suggests possible factors of ($x \pm 1$), ($x \pm 2$), ($x \pm 4$), ($x \pm 8$) and ($x \pm 16$)
But we already know that $x = 2$ is a solution !
Thus;

$$x^3 - 12x + 16 = (x - 2)(x^2 + 2x - 8)$$
$$= (x - 2)\ (x + 4)\ (x - 2)$$
$$= (x - 2)^2 (x + 4)$$

In fact, because there was a tangent at $x = 2$, it followed that the ($x - 2$) factor would be squared. We were after the other point of intersection from $x = -4$

i.e. $(-4, -60)$

[4 marks]