Grade Gobbler 7

Answer 1

(i)

$$x^2 + y^2 + 6x - 14y = 6$$

$$\left[x^2 + 6x \right] + \left[y^2 - 14y \right] = 6$$

$$\left[(x + 3)^2 - 9 \right] + \left[(y - 7)^2 - 49 \right] = 6$$

$$(x + 3)^2 + (y - 7)^2 = 8^2$$

[2 marks]

(ii)

$$V = \frac{\pi}{3} \times 64 \times 13.5$$

$$= 904.8 \text{ cm}^3$$

Answer 2

$$\left(\frac{1}{3x} + x \right)^4 = (1) \times \left(\frac{1}{3x} \right)^4 \times (x)^0$$

$$+ (4) \times \left(\frac{1}{3x} \right)^3 \times (x)^1$$

$$+ (6) \times \left(\frac{1}{3x} \right)^2 \times (x)^2$$

$$+ (4) \times \left(\frac{1}{3x} \right)^1 \times (x)^3$$

$$+ (1) \times \left(\frac{1}{3x} \right)^0 \times (x)^4$$

$$= \frac{1}{81x^4} + \frac{4}{27x^2} + \frac{2}{3} + \frac{4x^2}{3} + x^4$$

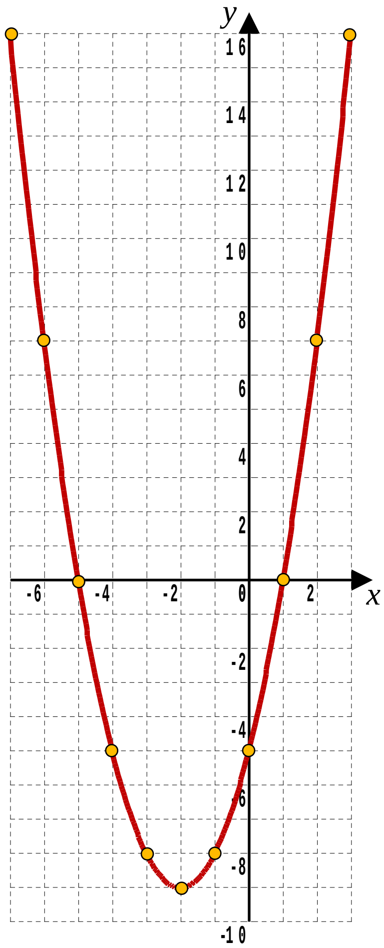
Answer 3

(i)

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
x^2	49	36	25	16	9	4	1	0	1	4	9
$4x - 5$	-33	-29	-25	-21	-17	-13	-9	-5	-1	3	7
y	16	7	0	-5	-8	-9	-8	-5	0	7	16

[2 marks]

(ii)



[2 marks]

(iii) $x^2 + 4x - 5 = (x + 5)(x - 1)$

[1 mark]

(iv) Crosses the x-axis when $x = -5$, $x = 1$

[1 mark]

(v) Crosses x-axis when $y = 0$.

When $(x + 5)(x - 1) = 0$ either $x = -5$ or $x = 1$

[1 mark]

(vi) $-5 < x < 1$

[2 marks]

Answer 4

$$\text{LHS} = \frac{\sqrt{1 - cos^2\,\theta}}{\sqrt{1 - sin^2\,\theta}}$$

$$= \frac{\sqrt{sin^2\,\theta}}{\sqrt{cos^2\,\theta}}$$

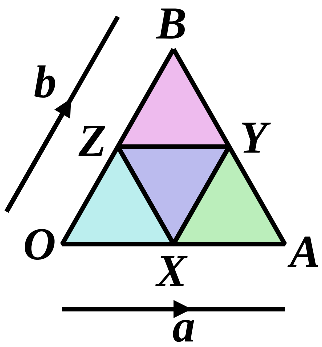
$$= \frac{sin\,\theta}{cos\,\theta}$$

$$= tan\,\theta$$

$$= \text{RHS} \qquad \square$$

[2 marks]

Answer 5



(i) $\overrightarrow{OX} = \frac{1}{2}\boldsymbol{a}$ **(ii)** $\overrightarrow{AX} = -\frac{1}{2}\boldsymbol{a}$ **(iii)** $\overrightarrow{ZB} = \frac{1}{2}\boldsymbol{b}$

(iv) $\overrightarrow{YZ} = -\frac{1}{2}\boldsymbol{a}$ **(v)** $\overrightarrow{ZO} = -\frac{1}{2}\boldsymbol{b}$ **(vi)** $\overrightarrow{BA} = \boldsymbol{a} - \boldsymbol{b}$

(vii) $\overrightarrow{ZX} = \frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

(viii) $\overrightarrow{OY} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

(ix) $\overrightarrow{AZ} = -\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

[3 marks]

Answer 6

$$y = \frac{4\,x^3 + 3\,x^5}{x^2}$$

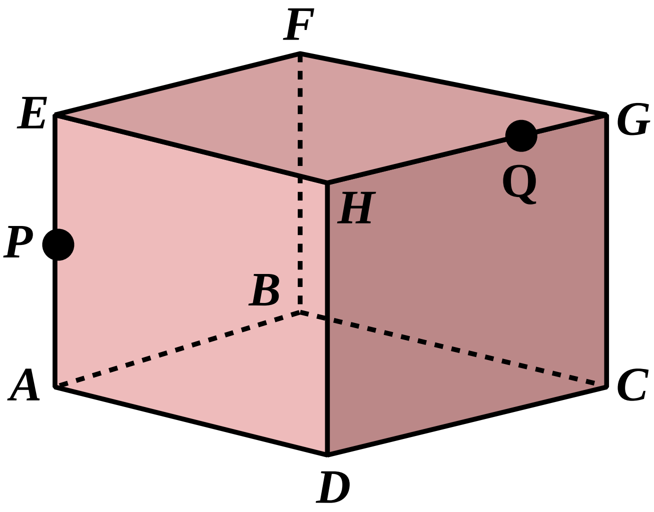
$$= \frac{4\,x^3}{x^2} + \frac{3\,x^5}{x^2}$$

$$= 4x + 3\,x^3$$

$$\frac{dy}{dx} = 4 + 9\,x^2$$

[2 marks]

Answer 7



(i) $|EQ| = \sqrt{4^2 + 3^2}$ $\quad |PD| = \sqrt{2^2 + 4^2}$

$= 5 \text{ cm}$ $\quad = 2\sqrt{5} \text{ cm}$ (About 4.472 cm)

[2 marks]

(ii)

$$|PQ| = \sqrt{2^2 + 5^2}$$

$$= \sqrt{29} \text{ cm} \qquad \text{(About 5.385 cm)}$$

[1 mark]

(iii) the angle between *PQ* and the plane *EFGH* is $\angle PQE$

$$\angle PQE = arctan\left(\frac{2}{\sqrt{29}}\right)$$

$$= 20.4°$$

[2 marks]

(iv) For angle *QPD* first use Pythagoras' Theorem, then the Cosine Rule backwards on triangle *QPD*

$$|QD| = \sqrt{4^2 + 3^2}$$

$$= 5 \text{ cm}$$

$$cos\ P = \frac{q^2 + d^2 - p^2}{2qd}$$

$$= \frac{(2\sqrt{5})^2 + (\sqrt{29})^2 - 5^2}{2 \times 2\sqrt{5} \times \sqrt{29}}$$

$$= \frac{20 + 29 - 25}{4\sqrt{145}}$$

$$= 0.4983$$

$$P = 60.1°$$

[3 marks]

Answer 8

(i) $1296 = 9 \times 144 \quad \therefore$ 144 is a factor of 1296

[1 mark]

(ii)

$$y = 36x^4 - 648x^2 + 18$$

$$\frac{dy}{dx} = 144x^3 - 1296x$$

$$= 144x(x^2 - 9)$$

[1 mark]

(iii) $\frac{dy}{dx} = 0$ when $x = 0$ or $x = \pm 3$

Turning Points : $(0,\ 18)$ $(\pm 3, -2898)$

[3 marks]

(iv)

$$\frac{d^2y}{dx^2} = 432x^2 - 1296$$

[1 mark]

(v) When $x = 0$ $\quad \frac{d^2y}{dx^2}$ is $-ve$ $\quad \therefore$ $(0,\ 0)$ is a local minimum

When $x = \pm 3$ $\quad \frac{d^2y}{dx^2}$ is $+ve$ $\quad \therefore$ $(\pm 3, -2898)$ are local maxima

[2 marks]

Answer 9

$$\int_0^1 (x + 5)(2x^2 + 1)\ dx = \int_0^1 2x^3 + 10x^2 + x + 5\ dx$$

$$= \left[\frac{x^4}{2} + \frac{10x^3}{3} + \frac{x^2}{2} + 5x \right]_0^1$$

$$= 9\frac{1}{3}$$

[2 marks]