

Answers to Grade Gobbler 9

Answer 1

$$\begin{aligned}y &= \frac{x^2 + 4x^5}{x^3} \\&= \frac{x^2}{x^3} + \frac{4x^5}{x^3} \\&= x^{-1} + 4x^2 \\ \frac{dy}{dx} &= -x^{-2} + 8x \\&= 8x - \frac{1}{x^2}\end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned}(\text{i}) \quad \left(2x + \frac{1}{x}\right)^4 &= (1) \times (2x)^4 \times \left(\frac{1}{x}\right)^0 \\&\quad + (4) \times (2x)^3 \times \left(\frac{1}{x}\right)^1 \\&\quad + (6) \times (2x)^2 \times \left(\frac{1}{x}\right)^2 \\&\quad + (4) \times (2x)^1 \times \left(\frac{1}{x}\right)^3 \\&\quad + (1) \times (2x)^0 \times \left(\frac{1}{x}\right)^4 \\&= 16x^4 + 32x^2 + 24 + \frac{8}{x^2} + \frac{1}{x^4}\end{aligned}$$

$$(\text{ii}) \quad 24$$

[1 mark]

$$(\text{iii}) \quad 8$$

[1 mark]

Answer 3

$$\begin{aligned}
 \text{(i)} \quad \text{Radius} &= \frac{1}{2} \times \sqrt{(\Delta x)^2 + (\Delta y)^2} \\
 &= \frac{1}{2} \times \sqrt{(51 - 3)^2 + (19 - 5)^2} \\
 &= \frac{1}{2} \times \sqrt{48^2 + 14^2} \\
 &= 25
 \end{aligned}$$

[2 marks]

$$\begin{aligned}
 \text{(ii)} \quad \text{Midpoint} &= \left(\frac{3 + 51}{2}, \frac{5 + 19}{2} \right) \\
 &= (27, 12) \quad \text{which is the centre of the circle}
 \end{aligned}$$

[2 marks]

$$\begin{aligned}
 \text{(iii)} \quad &(x - 27)^2 + (y - 12)^2 = 625 \\
 \text{i.e.} \quad &p = -27, \quad q = -12 \quad \text{and} \quad k = 625
 \end{aligned}$$

[2 marks]**Answer 4**

$$\text{(i)} \quad \text{twice}$$

[1 mark]

$$\begin{aligned}
 \text{(ii)} \quad P(3 \text{ fours}) &= {}^{12}C_3 \left(\frac{1}{6} \right)^3 \left(\frac{5}{6} \right)^9 \\
 &= 0.197 \, 396
 \end{aligned}$$

[3 marks]

Answer 5

(i) $f(x) = 0 \Rightarrow x^2 - 2x - 15 = 0$

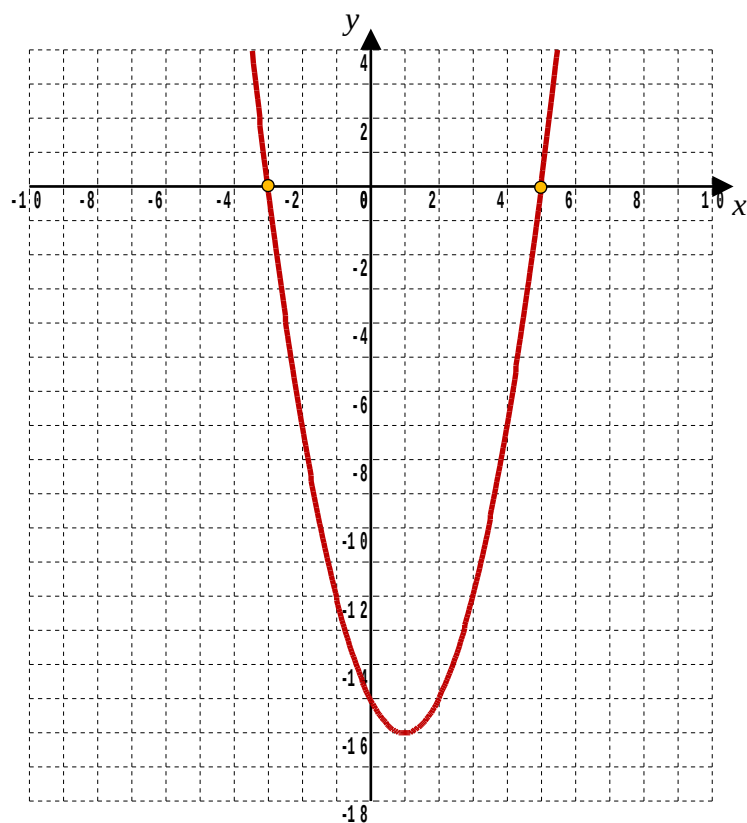
$$(x + 3)(x - 5) = 0$$

Either $(x + 3) = 0$ or $(x - 5) = 0$

$\therefore$ Crosses the x -axis at $(-3, 0)$, $(5, 0)$

[1 mark]

(ii)



[1 mark]

(iii) $-3 < x < 5$

[2 marks]

(iv) $x \leq -3$ or $x \geq 5$

[2 marks]

Answer 6

$$\begin{aligned}
\int_1^5 x + \frac{4}{x^2} dx &= \int_1^5 x + 4x^{-2} dx \\
&= \left[\frac{x^2}{2} - 4x^{-1} \right]_1^5 \\
&= \left[\frac{x^2}{2} - \frac{4}{x} \right]_1^5 \\
&= \left(\frac{25}{2} - \frac{4}{5} \right) - \left(\frac{1}{2} - \frac{4}{1} \right) \\
&= \frac{76}{5} \quad \text{or} \quad 15.2
\end{aligned}$$

[4 marks]**Answer 7**

(i) $y = -\frac{1}{2}x + 6$ has a gradient of $-\frac{1}{2}$

Perpendicular gradient: “sign changed reciprocal” i.e. 2

$$y = mx + c \Rightarrow 11 = 2 \times 5 + c \quad \therefore c = 1$$

$$y = 2x + 1 \text{ is the required line}$$

[2 marks]

(ii) $2x + 1 = -\frac{1}{2}x + 6$

$$x = 2 \text{ so } (2, 5) \text{ is the point of intersection}$$

[2 marks]**Answer 8**

By the cosine Rule,

$$\begin{aligned}
c^2 &= a^2 + b^2 - 2ab \cos C \\
&= 23^2 + 14^2 - 2 \times 23 \times 14 \times \cos 54 \\
&= 346.466 \\
c &= 18.6 \text{ cm}
\end{aligned}$$

[3 marks]

Answer 9

$$y = \frac{1}{2}x^4 - 4x^2 + 3$$

$$\frac{dy}{dx} = 2x^3 - 8x$$

$$\frac{dy}{dx} = 0 \quad \text{when} \quad 2x^3 - 8x = 0$$

$$\text{i.e. when } 2x(x^2 - 4) = 0$$

$$2x(x - 2)(x + 2) = 0$$

$$\text{Either } x = 0 \quad \text{or} \quad x = \pm 2$$

$$(0, 3), (\pm 2, -5)$$

$$\frac{d^2y}{dx^2} = 6x^2 - 8$$

$$(0, 3) \Rightarrow \frac{d^2y}{dx^2} \text{ is negative} \Rightarrow \text{Local MAXimum}$$

$$(\pm 2, -5) \Rightarrow \frac{d^2y}{dx^2} \text{ is positive} \Rightarrow \text{Local MINimum}$$

[4 marks]

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