Answers to Grade Gobbler 9

Answer 1

$$\begin{aligned} y &= \frac{x^2 + 4x^5}{x^3} \\ &= \frac{x^2}{x^3} + \frac{4x^5}{x^3} \\ &= x^{-1} + 4x^2 \\ \frac{dy}{dx} &= -x^{-2} + 8x \\ &= 8x - \frac{1}{x^2} \end{aligned}$$

[3 marks]

Answer 2

(i) $$\left(2x + \frac{1}{x}\right)^4 = (1) \times (2x)^4 \times \left(\frac{1}{x}\right)^0 + (4) \times (2x)^3 \times \left(\frac{1}{x}\right)^1 + (6) \times (2x)^2 \times \left(\frac{1}{x}\right)^2 + (4) \times (2x)^1 \times \left(\frac{1}{x}\right)^3 + (1) \times (2x)^0 \times \left(\frac{1}{x}\right)^4$$

$$= 16x^4 + 32x^2 + 24 + \frac{8}{x^2} + \frac{1}{x^4}$$

(ii) 24

[1 mark]

(iii) 8

[1 mark]

Answer 3

(i) $$Radius = \frac{1}{2} \times \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$= \frac{1}{2} \times \sqrt{(51 - 3)^2 + (19 - 5)^2}$$

$$= \frac{1}{2} \times \sqrt{48^2 + 14^2}$$

$$= 25$$

[2 marks]

(ii) $$Midpoint = \left(\frac{3 + 51}{2}, \frac{5 + 19}{2} \right)$$

$= (27,\ 12)$ which is the centre of the circle

[2 marks]

(iii) $$(x - 27)^2 + (y - 12)^2 = 625$$

i.e. $p = -27$, $q = -12$ and $k = 625$

[2 marks]

Answer 4

(i) twice

[1 mark]

(ii) $$P(3 fours) = {}^{12}C_3 \left(\frac{1}{6} \right)^3 \left(\frac{5}{6} \right)^9$$

$$= 0.197\,396$$

[3 marks]

Answer 5

(i) $f(x) = 0 \Rightarrow x^2 - 2x - 15 = 0$

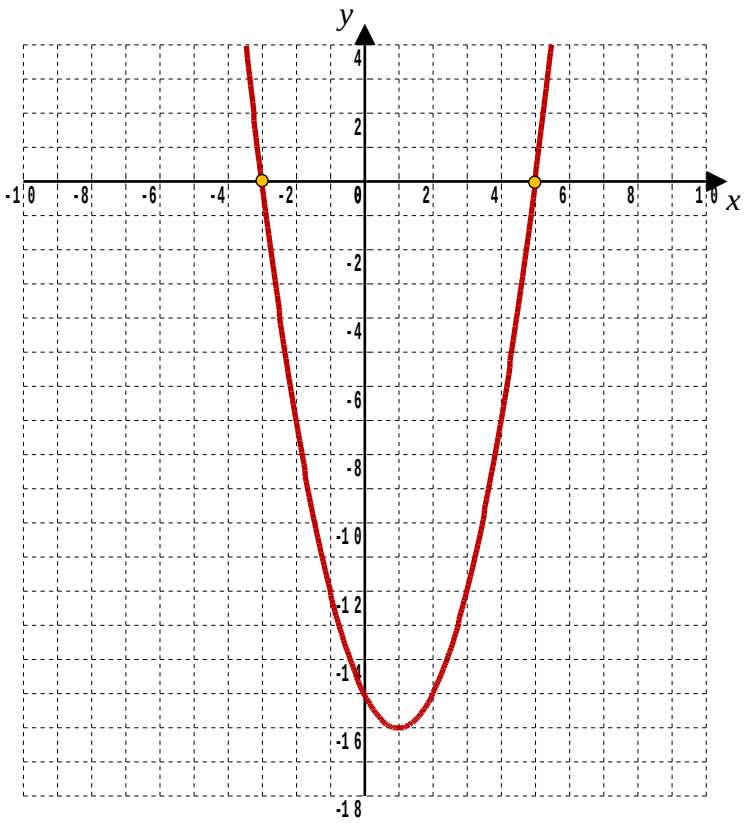
$(x + 3)(x - 5) = 0$

Either $(x + 3) = 0$ or $(x - 5) = 0$

$\therefore$ Crosses the x-axis at $(-3, 0)$, $(5, 0)$

[1 mark]

(ii)



[1 mark]

(iii) $-3 < x < 5$

[2 marks]

(iv) $x \leqslant -3$ or $x \geqslant 5$

[2 marks]

Answer 6

$$\int_1^5 x + \frac{4}{x^2}\, dx = \int_1^5 x + 4x^{-2}\, dx$$

$$= \left[\frac{x^2}{2} - 4x^{-1} \right]_1^5$$

$$= \left[\frac{x^2}{2} - \frac{4}{x} \right]_1^5$$

$$= \left(\frac{25}{2} - \frac{4}{5} \right) - \left(\frac{1}{2} - \frac{4}{1} \right)$$

$$= \frac{76}{5} \quad \text{or} \quad 15.2$$

[4 marks]

Answer 7

(i) $y = -\frac{1}{2}x + 6$ has a gradient of $-\frac{1}{2}$

Perpendicular gradient: "sign changed reciprocal" i.e. 2

$y = mx + c \Rightarrow 11 = 2 \times 5 + c \quad \therefore \; c = 1$

$y = 2x + 1$ is the required line

[2 marks]

(ii) $2x + 1 = -\frac{1}{2}x + 6$

$x = 2$ *so* $(2,\ 5)$ is the point of intersection

[2 marks]

Answer 8

By the cosine Rule,

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$= 23^2 + 14^2 - 2 \times 23 \times 14 \times \cos 54$$

$$= 346.466$$

$$c = 18.6 \text{ cm}$$

[3 marks]

Answer 9

$$y = \frac{1}{2}x^4 - 4x^2 + 3$$

$$\frac{dy}{dx} = 2x^3 - 8x$$

$$\frac{dy}{dx} = 0 \quad \text{when} \quad 2x^3 - 8x = 0$$

$$\text{i.e. when } 2x\left(x^2 - 4\right) = 0$$

$$2x\,(x-2)(x+2) = 0$$

$$\text{Either } x = 0 \quad \text{or} \quad x = \pm 2$$

$$(0,\ 3),\quad (\pm 2, -5)$$

$$\frac{d^2y}{dx^2} = 6x^2 - 8$$

$$(0,\ 3) \Rightarrow \frac{d^2y}{dx^2} \text{ is negative} \Rightarrow \text{Local MAXimum}$$

$$(\pm 2, -5) \Rightarrow \frac{d^2y}{dx^2} \text{ is positive} \Rightarrow \text{Local MINimum}$$

[4 marks]