Grade Gobbler
R E V I S I O N
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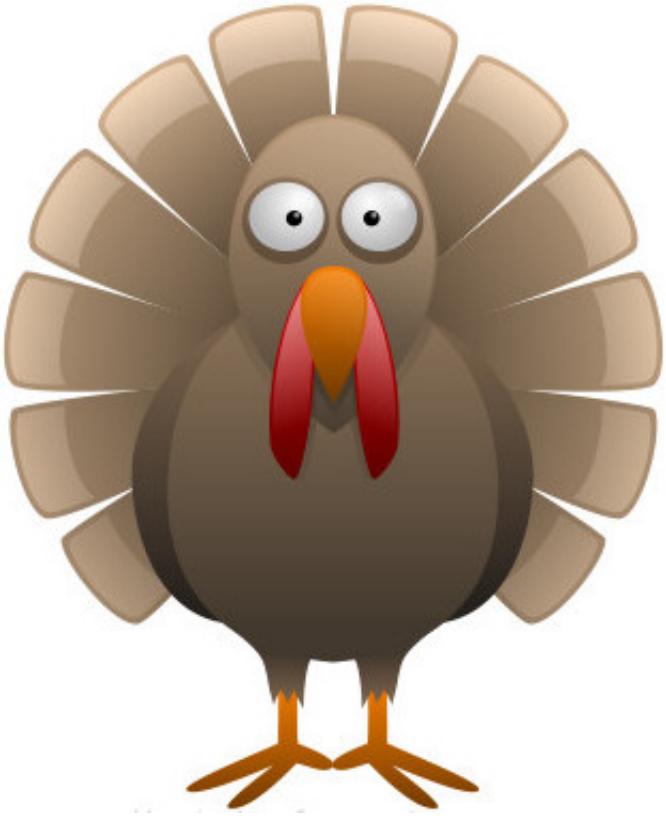

Additional Mathematics

FSMQ Mathematics

Additional Mathematics
Grade Gobbler 1

40 Mark Paper

Question 1

Determine the value of $\int_2^3 x^3 + 7 \ dx$

[4 marks]

Question 2

$$f(x) = x^4 + \frac{1}{x^2}$$

(i) State the value of $f(2)$

[1 mark]

(ii) Use the fact that $\frac{1}{x^2} = x^{-2}$ to help in finding the derivative of $f(x)$

[2 marks]

(iii) Find the value of $f'(2)$

[1 mark]

Question 3

(i) Use Pascal's triangle to expand the brackets;

$$(2 + 3x)^4$$

Give each term in your expansion in as simple a form as possible.

The following may be of help in setting out your working....

$$(2 + 3x)^4 = \left(\quad\right) \times \left(\quad\right) \times \left(\quad\right)$$

$$+ \left(\quad\right) \times \left(\quad\right) \times \left(\quad\right)$$

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$$+ \left(\quad\right) \times \left(\quad\right) \times \left(\quad\right)$$

[4 marks]

(ii) By using your part (i) answer, or otherwise, expand the brackets of;

$$\left(2 + 3\sqrt{x}\right)^4$$

[2 marks]

Question 4

(i) Use the factor theorem to prove that $(x - 3)$ is a factor of

$$f(x) = x^3 + 7x^2 - 6x - 72$$

[2 marks]

(ii) Hence, or otherwise, factorise completely the cubic polynomial.

[3 marks]

Question 5

A particle moves in a straight line with constant acceleration.
Initially its velocity is 2 m s^{-1} and after 5 seconds its velocity is 19 m s^{-1}
Find the acceleration.

[4 marks]

Question 6

By first expanding the brackets, determine the integral of;

$$y = \sqrt{x} + 4\,(x + 2)\,(x - 3)$$

[5 marks]

Question 7

$$f(x) = x^3 - 3x + 1$$

(i) Find all turning points of this function by solving the equation,

$$f'(x) = 0$$

[5 marks]

(ii) For each turning point clearly state, with justification, if it is a local minimum or a local maximum.

[3 marks]

Question 8

Provide proof that determines if a Δ with the following sides is right angled, or not,

$$\left(m^2 - n^2\right) \qquad 2mn \qquad \left(m^2 + n^2\right)$$

where m and n are positive integers with $m > n$.

[4 marks]

Grade Gobbler 1

Answer 1

$$\int_2^3 x^3 + 7 \; dx = \left[\frac{x^4}{4} + 7x \right]_2^3$$

$$= \left(\left[\frac{81}{4} + 7 \times 3 \right] - \left[4 + 7 \times 2 \right] \right)$$

$$= \frac{93}{4} \text{ or } 23.25$$

You will get NO MARKS in this question if you use the "integration" button on a calculator to get this numerical solution and have no working.

[4 marks]

Answer 2

(i) 16.25

[1 mark]

(ii) $f(x) = x^4 + x^{-2}$

$$\therefore \; f'(x) = 4x^3 - 2x^{-3}$$

$$= 4x^3 - \frac{2}{x^3}$$

[2 marks]

(iii) 31.75

[1 mark]

Answer 3

(i)

$$(2 + 3x)^4 = (1) \times (2)^4 \times (3x)^0$$
$$+ (4) \times (2)^3 \times (3x)^1$$
$$+ (6) \times (2)^2 \times (3x)^2$$
$$+ (4) \times (2)^1 \times (3x)^3$$
$$+ (1) \times (2)^0 \times (3x)^4$$
$$= 16 + 96x + 216x^2 + 216x^3 + 81x^4$$

[4 marks]

(ii) $\left(2 + 3\sqrt{x}\right)^4 = 16 + 96\sqrt{x} + 216x + 216\left(\sqrt{x}\right)^3 + 81x^2$

[2 marks]

Answer 4

(i)

$$f(3) = 3^3 + 7 \times 3^2 - 6 \times 3 - 72$$
$$= 27 + 63 - 18 - 72$$
$$= 0 \quad \therefore (x - 3) \text{ is a factor, by the factor theorem} \; \square$$

[2 marks]

(ii) $x^3 + 7x^2 - 6x - 72 = (x - 3)(x^2 + 10x + 24)$

$= (x - 3)(x + 4)(x + 6)$

[3 marks]

Answer 5

$$v = u + at$$
$$19 = 2 + a \times 5$$
$$17 = 5a$$
$$a = 3.4 \text{ m s}^{-2}$$

[4 marks]

Answer 6

$$\int \sqrt{x} + 4(x + 2)(x - 3)\,dx = \int x^{\frac{1}{2}} + 4(x^2 - x - 6)\,dx$$

$$= \int x^{\frac{1}{2}} + 4x^2 - 4x - 24\,dx$$

$$= \frac{2x^{\frac{3}{2}}}{3} + \frac{4x^3}{3} - 2x^2 - 24x + c$$

[5 marks]

Answer 7

(i) $f(x) = x^3 - 3x + 1$

$\Rightarrow \quad f'(x) = 3x^2 - 3$

We require $3x^2 - 3 = 0$

$x^2 = 1$ so $x = \pm 1$

Turning points are $(-1, +3), (1, -1)$

[5 marks]

(ii) $f''(x) = 6x$ is the "bend detector" equation

$(-1, +3)$ is a local maximum as $f''(-1)$ is negative

$(1, -1)$ is a local minimum as $f''(1)$ is positive

[3 marks]

Answer 8

The triangle will be right angled as Pythagoras' theorem holds.

i.e. $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$

which is proven by expanding all brackets to show LHS = RHS.

This relationship gives a method of finding Pythagorean Integer Triples.

[4 marks]

FSMQ Mathematics

Additional Mathematics
Grade Gobbler 2

40 Mark Paper

Question 1

Write each of the following in the form, $k\sqrt{2}$, where k is an integer.

(i) $\sqrt{50}$ **(ii)** $\sqrt{6} \times \sqrt{48}$ **(iii)** $\frac{14}{\sqrt{2}}$

[3 marks]

Question 2

Determine the value of, $\int_1^3 \frac{x^3 + 3x^2}{2x}\, dx$

Hint : Wedge it !

[5 marks]

Question 3

(a) Use the ${}^{n}C_{r}$ button on your calculator to determine the values of,

(i) ${}^{5}C_{2}$ **(ii)** ${}^{8}C_{3}$ **(iii)** ${}^{15}C_{5}$

[3 marks]

(b) Show where ${}^{5}C_{2}$ is on a diagram of Pascal's Triangle.

[2 marks]

(c) A coin is biased such that, when tossed, the probability of it landing heads is 0.4. This coin is tossed ten times.
Use the formula;

$$P(r \text{ heads}) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

to calculate the probability that exactly four of the ten tosses are heads.
Give your answer to three decimal places.

In this formula,

n is the number of times the coin is tossed

r is the number of heads

p is the probability of heads occurring

q is the probability of heads NOT occurring

[4 marks]

Question 4

A straight line has equation $y = 2x + 4$

A circle has equation $(x + 1)^2 + (y - 3)^2 = 10$

(i) Write down the coordinates of the centre of the circle.

[2 marks]

(ii) Determine, accurate to two decimal places, the radius of the circle.

[1 mark]

(iii) Find the coordinates of the two points where the line and circle intersect.

[6 marks]

Question 5

A rocket is catapulted vertically upwards from its launch pad.

Its height in metres, h, at time, t seconds, is given by;

$$h = 0.5t^3 - 4t^2 + 10t$$

(i) How high is the rocket when $t = 2$?

[1 mark]

(ii) Find $\frac{dh}{dt}$, an expression for the vertical velocity of the rocker at time t.

[2 marks]

(iii) With what vertical velocity was the rocket launched ?

[1 mark]

(iv) Find $\frac{d^2h}{dt^2}$, an expression for the vertical acceleration at time t.

[1 mark]

The rocket, as it travels upwards, is initially slowing down until the weight of fuel burnt makes it light enough to start accelerating upwards.

(v) For how long is the rocket's acceleration upwards negative ?

[3 marks]

Question 6

Expand the brackets, $(x - 1)(x + 1)^3$

[6 marks]

Answers to Grade Gobbler 2

Answer 1

(i) $5\sqrt{2}$ **(ii)** $12\sqrt{2}$ **(iii)** $7\sqrt{2}$

[3 marks]

Answer 2

$$\int_1^3 \frac{x^3 + 3x^2}{2x}\,dx = \int_1^3 \frac{x^3}{2x} + \frac{3x^2}{2x}\,dx$$

$$= \int_1^3 \frac{x^2}{2} + \frac{3x}{2}\,dx$$

$$= \left[\frac{x^3}{6} + \frac{3x^2}{4}\right]_1^3$$

$$= \left(\left[\frac{27}{6} + \frac{27}{4}\right] - \left[\frac{1}{6} + \frac{3}{4}\right]\right)$$

$$= 10\frac{1}{3}$$

You will get NO MARKS in this question if you use the "integration" button on a calculator to get this numerical solution and have no working.

[5 marks]

Answer 3

(a) **(i)** 10

(ii) 56

(iii) 3003

[3 marks]

(b)

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 * **10** * 10 5 1

[2 marks]

(c)

$$P(4\,heads) = {}^{10}C_4\ 0.4^4\ 0.6^6$$

$$= 210 \times 0.0256 \times 0.046656$$

$$= 0.251$$

[4 marks]

Answer 4

(i) $(-1,\ 3)$

[2 marks]

(ii) 3.16

[1 mark]

(iii)

$$(x + 1)^2 + (y - 3)^2 = 10$$

$$(x + 1)^2 + (2x + 4 - 3)^2 = 10$$

$$(x + 1)^2 + (2x + 1)^2 = 10$$

$$x^2 + 2x + 1 + 4x^2 + 4x + 1 = 10$$

$$5x^2 + 6x - 8 = 10$$

$$(5x - 4)(x + 2) = 0$$

$$\text{Either } x = \frac{4}{5} \text{ or } x = -2$$

$$\text{Points are } \left(\frac{4}{5},\ \frac{28}{5}\right),\quad (-2,\ 0)$$

[6 marks]

Answer 5

(i) 8 metres

[1 mark]

(ii) $\dfrac{dh}{dt} = 1.5t^2 - 8t + 10$

[2 marks]

(iii) 10 m s^{-1}

(iv) $\dfrac{d^2h}{dt^2} = 3t - 8$

[1 mark]

(v) 2.67 seconds

[3 marks]

Answer 6

$$\begin{aligned}(x - 1)(x + 1)^3 &= (x - 1)(1 + 3x + 3x^2 + x^3)\\ &= x + 3x^2 + 3x^3 + x^4 - 1 - 3x - 3x^2 - x^3\\ &= x^4 + 2x^3 - 2x - 1\end{aligned}$$

Additional Mathematics
Grade Gobbler 3

40 Mark Paper

Question 1

The circumference of a circle with centre $(8, -15)$ passes through the origin.

(i) What is the radius of the circle ?

[2 marks]

(ii) Write down the equation of the circle.

[2 marks]

Question 2

Use Pascal's triangle to expand the brackets;

$$(5 - 2x)^3$$

Give each term in your expansion in as simple a form as possible.

The following may be of help in setting out your working....

$$(5 - 2x)^3 = \left(\quad\right) \times \left(\quad\right) \times \left(\quad\right)$$
$$+ \left(\quad\right) \times \left(\quad\right) \times \left(\quad\right)$$
$$+ \left(\quad\right) \times \left(\quad\right) \times \left(\quad\right)$$
$$+ \left(\quad\right) \times \left(\quad\right) \times \left(\quad\right)$$

[4 marks]

Question 3

Solve the following equation for $0° \leqslant \theta \leqslant 360°$

$$2\sin^2\theta - \sin\theta - 1 = 0$$

[5 marks]

Question 4

$$f(x) = \sqrt{x}\left(1 + \sqrt{x}\right)$$

Find $f'(x)$

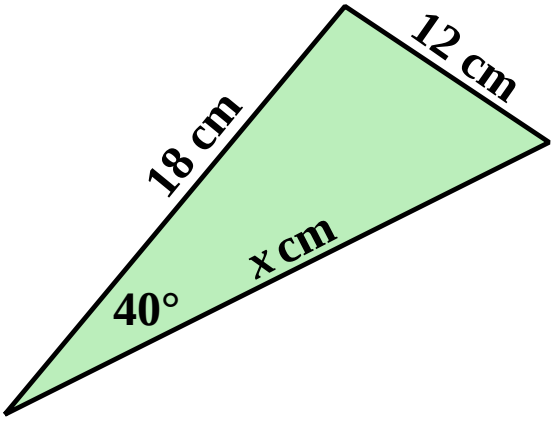
[3 marks]

Question 5

A car accelerates uniformly at 4 m s^{-2} for 12 seconds.
Initially, the car was moving at 13 m s^{-1}
What distance does the car cover during this period of acceleration ?

[3 marks]

Question 6



(i) Find the length of the side marked *x* in the diagram above.

[3 marks]

(ii) Find the area of the triangle.

[2 marks]

Question 7

Find, to 3 significant figures, the value of;

(i) $\sqrt[7]{\pi^3}$ **(ii)** $\left(\frac{1}{\pi}\right)^{\pi}$

[2 marks]

Question 8

A circle has equation $x^2 + y^2 - 10x - 8y + 16 = 0$

(i) Write the equation of this circle in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

where a, b and r are constants with values to be found.

[3 marks]

(ii) State the coordinates of the circle's centre, and its radius.

[2 marks]

(iii) Find the two points where the circle intersects in x-axis.

[2 marks]

(iv) Where does the circle intersect the y-axis ?

[1 marks]

(v) The point (9, 7) is on the circumference of the circle.
Find the equation of the tangent to the circle at (9, 7).

[4 marks]

Question 9

$$f(x) = x^3 + x$$

Prove that $f(x)$ has no turning points.

[3 marks]

Answers to Grade Gobbler 3

Answer 1

(i) By The Theorem of Pythagoras, 17

[2 marks]

(ii) $(x - 8)^2 + (y + 15)^2 = 17^2$

[2 marks]

Answer 2

$$\begin{aligned}(5 - 2x)^3 &= (1) \times (5)^3 \times (-2x)^0 \\ &\quad + (3) \times (5)^2 \times (-2x)^1 \\ &\quad + (3) \times (5)^1 \times (-2x)^2 \\ &\quad + (1) \times (5)^0 \times (-2x)^3 \\ &= 125 - 150x + 60x^2 - 8x^3\end{aligned}$$

[4 marks]

Answer 3

$$2\sin^2\theta - \sin\theta - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\text{Either} \quad \sin x = -\frac{1}{2} \qquad \text{or} \qquad \sin x = 1$$

$$x_{WA} = \arcsin\left(\frac{1}{2}\right) \qquad x = \arcsin(1)$$

$$x_{WA} = 30^\circ \qquad x = 90^\circ$$

$$x = \{90^\circ, 210^\circ, 330^\circ\}$$

[5 marks]

Answer 4

$$\begin{aligned}f(x) = \sqrt{x}\left(1 + \sqrt{x}\right) &= \sqrt{x} + x \\ &= x^{\frac{1}{2}} + x \\ \therefore f'(x) &= \frac{1}{2}x^{-\frac{1}{2}} + 1 \\ &= \frac{1}{2\sqrt{x}} + 1\end{aligned}$$

[3 marks]

Answer 5

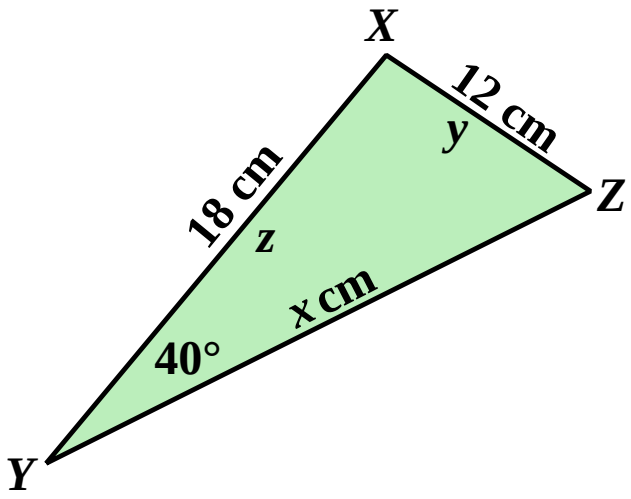
$$s = ut + \frac{1}{2}at^2$$

$$= 13 \times 12 + \frac{1}{2} \times 4 \times 12^2 \Rightarrow s = 444 \text{ metres}$$

[3 marks]

Answer 6

Label the triangle *XYZ*...



(i) By the inverted Sine Rule,

$$\frac{\sin Z}{18} = \frac{\sin 40}{12}$$

$$\sin Z = \frac{18 \sin 40}{12}$$

$$= 0.96418$$

$$Z = 74.62°$$

As the angles in a triangle sum to 180°,

$$\therefore X = 180 - 40 - 74.62$$

$$= 65.38°$$

Now to use the Sine Rule,

$$\frac{x}{\sin 65.38} = \frac{12}{\sin 40}$$

$$x = \frac{12 \sin 65.38}{\sin 40}$$

$$= 19.97 \text{ cm}$$

[3 marks]

(ii) Taking care to use the included angle,

$$Area \, \Delta = \frac{1}{2} x z \sin Y$$

$$= \frac{1}{2} \times 19.97 \times 18 \times \sin 40°$$

$$= 115.5 \text{ cm}^2$$

[2 marks]

Answer 7

(i) 1.63 **(ii)** 0.0274

[2 marks]

Answer 8

(i) $(x-5)^2 + (y-4)^2 = 5^2$

[3 marks]

(ii) Centre (5, 4) Radius 5

[2 marks]

(iii) When $y = 0$, $x^2 - 10x + 16 = 0$

$$(x-2)(x-8) = 0$$

$\therefore$ Crosses the x-axis at $(2, 0)$, $(8, 0)$

[2 marks]

(iv) When $x = 0$, $y^2 - 8y + 16 = 0$

$$(y-4)(y-4) = 0$$

$\therefore$ Crosses the y-axis at $(0, 4)$

[1 mark]

(v) Gradient between (5, 4) and (9, 7) will be...

$$m_r = \frac{7-4}{9-5} = \frac{3}{4}$$

So the gradient of the tangent, perpendicular to this radius will be,

$$m_t = -\frac{4}{3}$$

$$y = mx + c \quad \Rightarrow 7 = -\frac{4}{3} \times 9 + c \Rightarrow c = 19$$

The tangent will thus have the equation $y = -\frac{4}{3}x + 19$

[4 marks]

Answer 9

Trying to find turning points...

$$f'(x) = 0$$

$$3x^2 + 1 = 0$$

$$3x^2 = -1$$

$$x^2 = -\frac{1}{3}$$

$$x = \pm\sqrt{-\frac{1}{3}}$$ which has no real solutions

i.e. The curve has no turning points $\square$

[3 marks]

Additional Mathematics
Grade Gobbler 4

40 Mark Paper

Question 1

A straight line passes through the points (2, 5) and (8, 2)

(i) Find the equation of this line in the form $y = m\,x + c$

[2 marks]

A second straight line passes through the y-axis at (0, 4)

It is also perpendicular to the first line.

(ii) Find the equation of this second line, also in the form $y = m\,x + c$

[2 marks]

Question 2

The gradient of a curve is given by;

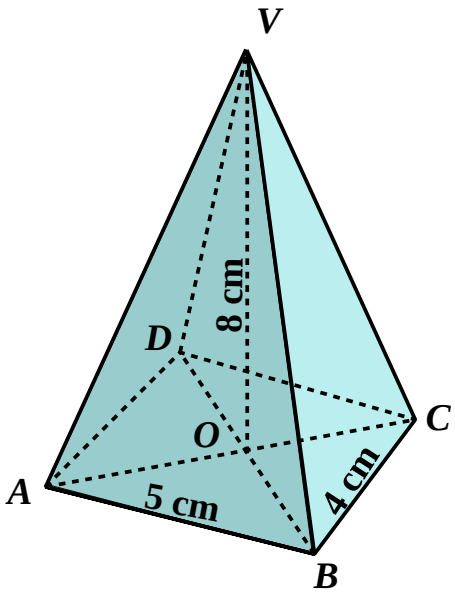
$$\frac{dy}{dx} = 4 + x - 3x^2$$

Find the equation of the curve given that it passes through the point (2, 5)

[4 marks]

Question 3

A pyramid *ABCDV* has a rectangular, horizontal, base *ABCD* of sides 5 cm and 4 cm.
The vertex *V*, is vertically above the centre of the base *O*.
The pyramid has height 8 cm.



Find the angle *VAC*.

[4 marks]

Question 4

Write the following as a single fraction in as simple a form as possible;

$$\frac{1}{(1+\sqrt{x})} + \frac{1}{(1-\sqrt{x})}$$

[4 marks]

Question 5

$$f(x) = x^3 - 5x^2 + 3x + 9$$

(i) Factorise $f(x)$ completely.

Hint : The factor theorem may help you get started.

[5 marks]

(ii) Hence, or otherwise, solve the equation $f(x) = 0$

[2 marks]

Question 6

Find the four values of θ in the range $0° \leqslant \theta \leqslant 360°$ that satisfy the equation;

$$cos\, 2\theta \;=\; 0.413$$

[5 marks]

Question 7

Expand the brackets, simplifying each term,

$$\left(x + \frac{5}{x} \right)^3$$

[4 marks]

Question 8

(i) Show that the point (2, 6) lies on the curve with equation

$$y = x^3 - x$$

[1 mark]

(ii) Find the equation of the tangent to the curve at the point (2, 6)

[3 marks]

(iii) Where does this tangent again meet the curve ?

[4 marks]

Answers to Grade Gobbler 4

Answer 1

(i)

$$m = \frac{\Delta y}{\Delta x} = \frac{2 - 5}{8 - 2} = \frac{-3}{6} = -\frac{1}{2}$$

$$y = mx + c \Rightarrow 5 = -\frac{1}{2} \times 2 + c \qquad \therefore c = 6$$

$$y = -\frac{1}{2}x + 6$$

[2 marks]

(ii)

Perpendicular gradient will be $+ 2$

$$y = 2x + 4$$

[2 marks]

Answer 2

$$y = \int 4 + x - 3x^2 \, dx$$

$$y = 4x + \frac{x^2}{2} - x^3 + c$$

Told that (2, 5) is on this curve...

$$5 = 4 \times 2 + \frac{2^2}{2} - 2^3 + c$$

$$\therefore c = 3$$

And the equation of the curve is thus;

$$y = 4x + \frac{x^2}{2} - x^3 + 3$$

[4 marks]

Answer 3

$$|AC| = \sqrt{4^2 + 5^2}$$

$$= \sqrt{41}$$

$$\angle VAC = \arctan\left(\frac{8}{\frac{1}{2}\sqrt{41}}\right)$$

$$= 68.2°$$

[4 marks]

Answer 4

$$\frac{1}{(1+\sqrt{x})} \times \frac{(1-\sqrt{x})}{(1-\sqrt{x})} + \frac{1}{(1-\sqrt{x})} \times \frac{(1+\sqrt{x})}{(1+\sqrt{x})}$$

$$= \frac{(1-\sqrt{x})+(1+\sqrt{x})}{(1-\sqrt{x})(1+\sqrt{x})}$$

$$= \frac{2}{1-x}$$

[4 marks]

Answer 5

(i) $f(x) = x^3 - 5x^2 + 3x + 9$

The 9 on the end suggests the possible factors are $(x \pm 1)$, $(x \pm 3)$ and $(x \pm 9)$

We're looking for a value of x that causes $f(x)$ to equal zero.

$f(1) = 8$ which is no good but $f(-1) = 0$ and so $(x+1)$ is a factor

Thus $x^3 - 5x^2 + 3x + 9 = (x+1)(x^2 - 6x + 9)$

$$= (x+1)(x-3)^2$$

[5 marks]

(ii) $x = -1,\ x = 3$ are the solutions

You will get NO MARKS in this question if you use the "solve equation" facility on a calculator to get these solutions without any working.

[2 marks]

Answer 6

$$\cos 2\theta = 0.413$$

$$2\theta = \arccos 0.413$$

$$2\theta = 65.6°,\ 294.4°,\ 425.6°,\ 654.4°$$

$$\theta = 32.8°,\ 147.2°,\ 212.8°,\ 327.2°$$

[5 marks]

Answer 7

$$\left(x + \frac{5}{x}\right)^3 = (1) \times (x)^3 \times \left(\frac{5}{x}\right)^0$$

$$+ (3) \times (x)^2 \times \left(\frac{5}{x}\right)^1$$

$$+ (3) \times (x)^1 \times \left(\frac{5}{x}\right)^2$$

$$+ (1) \times (x)^0 \times \left(\frac{5}{x}\right)^3$$

$$= x^3 + 15x + \frac{75}{x} + \frac{125}{x^3}$$

[4 marks]

Answer 8

(i)

$$y = x^3 - x$$

$$\text{When } x = 2, \quad y = 2^3 - 2$$

$$= 8 - 2$$

$$= 6$$

$$\therefore \ (2,\ 6) \text{ is on the curve} \qquad \square$$

[1 mark]

(ii)

$$\frac{dy}{dx} = 3x^2 - 1$$

$$\text{When } x = 2, \quad \frac{dy}{dx} = 11$$

$$y = mx + c \Rightarrow 6 = 11 \times 2 + c$$

$$\therefore c = -16$$

The equation of the tangent is $y = 11x - 16$

[3 marks]

(iii)

$$\text{Key step :} \qquad x^3 - x = 11x - 16$$

$$x^3 - 12x + 16 = 0$$

The 16 suggests possible factors of $(x \pm 1)$, $(x \pm 2)$, $(x \pm 4)$, $(x \pm 8)$ and $(x \pm 16)$
But we already know that $x = 2$ is a solution !
Thus;

$$x^3 - 12x + 16 = (x - 2)(x^2 + 2x - 8)$$

$$= (x - 2)\ (x + 4)\ (x - 2)$$

$$= (x - 2)^2 (x + 4)$$

In fact, because there was a tangent at $x = 2$, it followed that the $(x - 2)$ factor would be squared. We were after the other point of intersection from $x = -4$

$$\text{i.e. } (-4, -60)$$

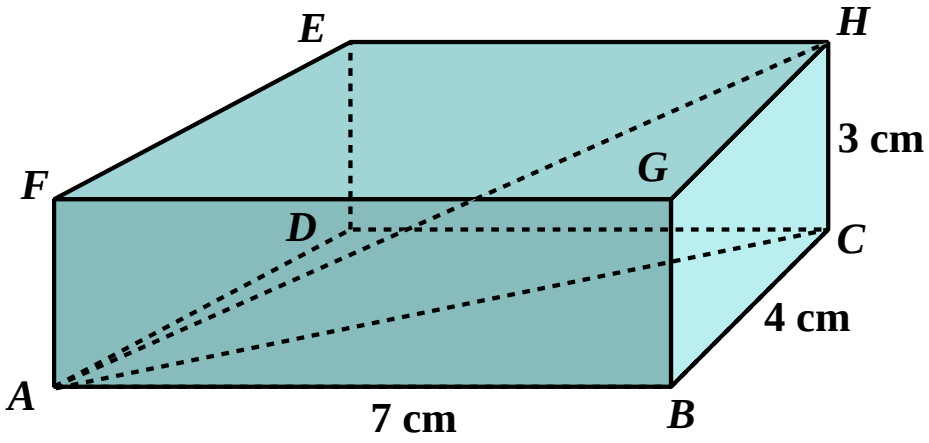
[4 marks]

Additional Mathematics
Grade Gobbler 5

40 Mark Paper

Question 1

A cuboid *ABCDEFGH* measures 7 cm by 4 cm by 3 cm, as shown below.



Calculate, in degrees, the angle *HAC*.
Give your answer accurate to 1 decimal place.

[3 marks]

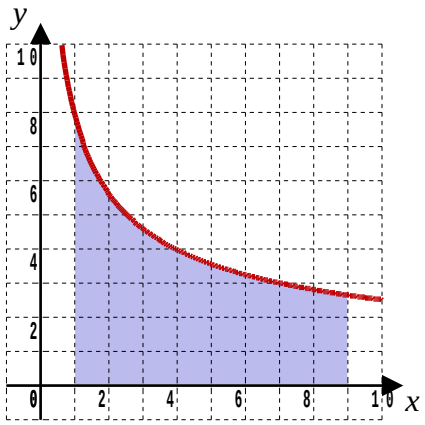
Question 2

$$f(x) = 1 + \frac{7x^2}{4} - x^3$$

Determine the value of $f'(3)$

[4 marks]

Question 3



Use integration to find, $\int_1^9 \frac{8}{\sqrt{x}}\, dx$

[4 marks]

Question 4

A circle has equation, $x^2 + y^2 - 8x + 2y + 8 = 0$

(i) Write the circle's equation in the form $(x - a)^2 + (y - b)^2 = r^2$ where a, b and r are constants to be found.

[3 marks]

(ii) Hence, or otherwise, state the coordinates of the circle's centre and its radius.

[2 marks]

Question 5

A quadratic equation of the form $a x^2 + bx + c = 0$

has solutions given by the well known formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(i) Show how to use this formula to find the solutions of;

$$x^2 + x - 1 = 0$$

Give your answers correct to three significant figures.

[4 marks]

The discriminant, D, is the piece of this formulae that is under the square root sign.

i.e. $D = b^2 - 4ac$

(ii) Calculate the discriminant associated with the quadratic equation;

$$x^2 + 8x + 26 = 0$$

[2 marks]

(iii) What does the part (ii) answer tell you about $x^2 + 8x + 26 = 0$?

[1 mark]

Question 6

(a) Use the ${}^{n}C_{r}$ button on your calculator to determine the values of,

(i) ${}^{7}C_{3}$ **(ii)** ${}^{11}C_{7}$ **(iii)** ${}^{16}C_{4}$

[3 marks]

(b) In a very large box of Smarties™, 14% are blue.
In a dark cinema, Charlie eats five Smarties™ without looking at their colour.
Consider the formula;

$$P(\,r \text{ Blue Smarties}^{TM}\,) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

In this formula,

n is the number of Smarties™ eaten

r is the number of Blue Smarties™ eaten

p is the probability of a Blue Smartie™ being eaten

q is the probability of a non-Blue Smartie™ being eaten

(i) Calculate the probability that exactly two of the five Smarties™ eaten are blue. Give your answer to three decimal places.

[2 marks]

(ii) Calculate the probability that exactly four of the five Smarties™ eaten is blue. Give your answer to six decimal places.

[2 marks]

(iii) What is the significance of the box of Smarties™ being described as *very large* ?

[1 mark]

Question 7

(a) $v^2 = u^2 + 2as$

Make *s* the subject of the formula.

[1 marks]

(b) A car crosses a speed hump with a velocity of 4 m s^{-1}
It then accelerates at a rate of 2.5 m s^{-2} to a speed of 9 m s^{-1} when the driver applies the brakes, causing an acceleration of – 3 m s^{-2}, reducing the speed of the car to 4 m s^{-1} to cross the next hump.

(i) How far apart are the humps ?

[2 marks]

(ii) How long does the car take to travel from one hump to the next ?

[2 marks]

(iii) The question implies that the car is being modelled as a particle.
In what way does this assumption affect your results ?

[1 mark]

Question 8

$$f(x) = 4x^4 + px^3 + x^2 - 2 \qquad \text{where } p \text{ is a constant}$$

When $f(x)$ is divided by $(2x - 1)$ the remainder is -1.75

By using the remainder theorem, or otherwise, determine the value of p.

[3 marks]

Answers to Grade Gobbler 5

Answer 1

$$|AC| = \sqrt{7^2 + 4^2} = \sqrt{65} \qquad \text{(About 8.06 cm)}$$

$$\angle HAC = \arctan\left(\frac{3}{\sqrt{65}}\right) = 20.4°$$

[4 marks]

Answer 2

$$f(x) = 1 + \frac{7x^2}{4} - x^3$$

$$f'(x) = \frac{7x}{2} - 3x^2$$

$$f'(3) = -16.5$$

[4 marks]

Answer 3

$$\int_1^9 \frac{8}{\sqrt{x}}\, dx = 8\int_1^9 8x^{-\frac{1}{2}}\, dx$$

$$= \left[16x^{\frac{1}{2}}\right]_1^9$$

$$= 32$$

[4 marks]

Answer 4

(i)

$$x^2 + y^2 - 8x + 2y + 8 = 0$$

$$\left[x^2 - 8x\right] + \left[y^2 + 2y\right] + 8 = 0$$

$$\left[(x-4)^2 - 16\right] + \left[(y+1)^2 - 1\right] + 8 = 0$$

$$(x-4)^2 + (y+1)^2 = 3^2$$

[3 marks]

(ii) Centre $(4, -1)$, Radius 3

[2 marks]

Answer 5

(i) Minimum working :

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = -1.62,\ 0.618$$

[4 marks]

(ii) $D = 8^2 - 4 \times 1 \times 26 = -40$

[2 marks]

(iii) The negative discriminant can't be square rooted over the reals.
$\therefore$ Quadratic has no solutions in Real numbers.

[1 mark]

Answer 6

(a) **(i)** ${}^7C_3 = 35$ **(ii)** ${}^{11}C_7 = 330$ **(iii)** ${}^{16}C_4 = 1820$

[3 marks]

(b) **(i)** $P(2\ blue) = {}^5C_2\, 0.14^2\, 0.86^3$

$= 0.125$

[2 marks]

(ii) $P(4\ blue) = {}^5C_4 \left(\frac{1}{13}\right)^4 \left(\frac{12}{13}\right)^1$

$= 0.001\,659$

[2 marks]

(iii) Eating a Smartie™ does not alter the 14% : 86% balance of probabilities.

[1 mark]

Answer 7

(a)
$$v^2 = u^2 + 2as$$
$$2as = v^2 - u^2$$
$$s = \frac{v^2 - u^2}{2a}$$

[1 mark]

(b) **(i)**
$$Hump\ separation = \frac{9^2 - 4^2}{2 \times 2.5} + \frac{4^2 - 9^2}{2 \times (-3)}$$
$$= 13 + \frac{65}{6} = \frac{143}{6} \text{ (About } 23.83 \text{ m)}$$

[2 marks]

(ii)
$$v = u + at \Leftrightarrow t = \frac{v - u}{a}$$
$$Total\ Time = \frac{9 - 4}{2.5} + \frac{4 - 9}{(-3)}$$
$$= 2 + \frac{5}{3} = 3.67 \text{ seconds}$$

[2 marks]

(iii) Various: e.g. Car has length, which a particle does not.

[1 mark]

Answer 8

By the remainder theorem $f\left(\frac{1}{2}\right) = -1.75$
$$4 \times \left(\frac{1}{2}\right)^4 + p\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 2 = -1.75$$
$$p = -2$$

Additional Mathematics
Grade Gobbler 6

40 Mark Paper

Question 1

Differentiate, $y = 5x^3 + \frac{4}{x^2}$

[3 marks]

Question 2

A function, f, is given by, $f(x) = x^3 + kx^2 + 3x - 10$

It is known that $(x - 1)$ is a factor of $f(x)$

(i) Determine the value of k.

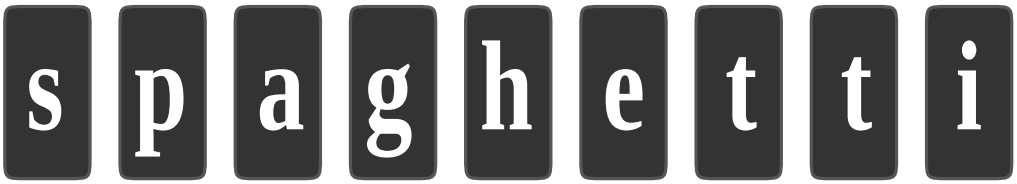
[1 mark]

(ii) Find, in terms of x, two more factors of $f(x)$

[3 marks]

Question 3

One letter is chosen at random from the word;



It is replaced, and then a second letter is chosen, again at random.
This process is repeated ten times in total.

Use the Binomial Theorem formula;

$$P(X = r) = {}^{n}C_r \, p^r \, q^{n-r} \qquad \text{(where } r = 0, 1, 2, ..., n\text{)}$$

to calculate the probability that exactly four of the ten choices are the letter "**t**"

[4 marks]

Question 4

A triangle has sides of lengths 4 cm, 7 cm and 10 cm
Find the size of the largest angle in the triangle.
Give your answer in degrees and accurate to one decimal place.

[3 marks]

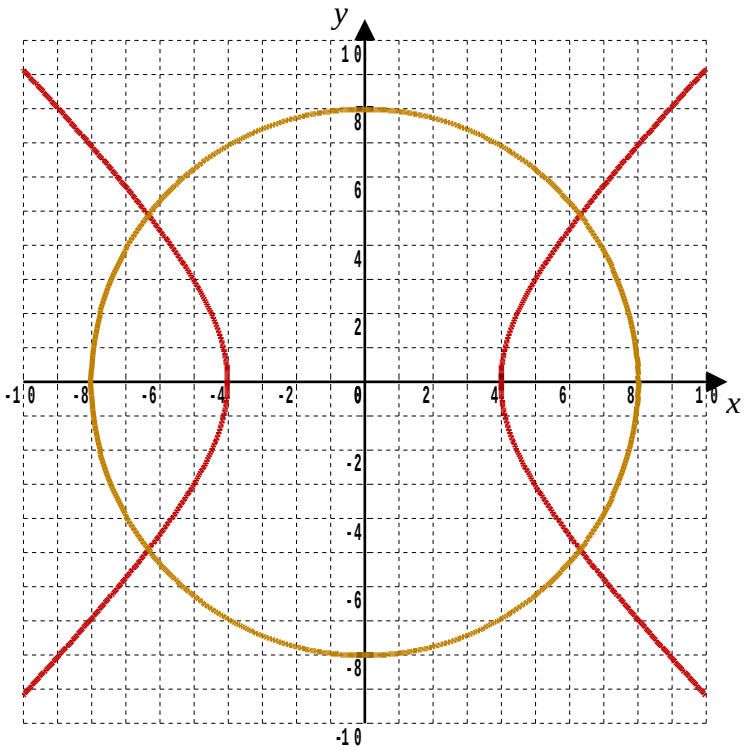
Question 5

Use integration to find the exact area bounded by the x-axis, the line $x = 1$, the line $x = 4$, and the curve $y = x^2 + 4$

[3 marks]

Question 6

The graph below is of the hyperbola, $x^2 - y^2 = 16$ and circle, $x^2 + y^2 = 64$



It can be seen from the graphs that the hyperbola and circle intersect at four points. Find the coordinates of these points of intersection.

[4 marks]

Question 7

Find all solutions in the interval $0° \leqslant x \leqslant 360°$ to the equation;

$$2\cos^2 x - 3\sin x = 3$$

[6 marks]

Question 8

Find an expression for $f^{-1}(x)$, given that $f(x) = \dfrac{4x + 5}{3}$

[3 marks]

Question 9

An truck moves with constant acceleration along a straight horizontal road.
The truck passes the point *A* with speed 5 m s^{-1}
It passes the point *B* 4 seconds later.
AB = 50 m

(a) Determine the acceleration of the truck.

[3 marks]

When the car passes the point *C*, it has speed 30 m s^{-1}

(b) Find the distance *AC*.

[3 marks]

Question 10

Find the equation of the tangent to the curve $y = x^3$ at the point $\left(\sqrt{2},\ 2\sqrt{2}\right)$.
Give your answer in the form $y = m\,x + k\sqrt{2}$ where *m*, and *k* are integers to be found.

[4 marks]

Answers to Grade Gobbler 6

Answer 1

$$\begin{aligned} y &= 5x^3 + \frac{4}{x^2} \\ &= 5x^3 + 4x^{-2} \\ \frac{dy}{dx} &= 15x^2 - 8x^{-3} \\ &= 15x^2 - \frac{8}{x^3} \end{aligned}$$

[3 marks]

Answer 2

(i) $f(1) = 0$ by the factor theorem

$$\begin{aligned} 1 + k + 3 - 10 &= 0 \\ \therefore\ k &= 6 \end{aligned}$$

[1 mark]

(ii) $x^3 + 6x^2 + 3x - 10 = (x - 1)(x^2 + 7x + 10)$

$$= (x - 1)(x + 5)(x + 3)$$

[3 marks]

Answer 3

$$\begin{aligned} P(4\ letter\ t) &= {}^{10}C_4 \left(\frac{2}{9}\right)^4 \left(\frac{7}{9}\right)^6 \\ &= 0.113\,371 \end{aligned}$$

[4 marks]

Answer 4

$$\begin{aligned} \cos A &= \frac{4^2 + 7^2 - 10^2}{2 \times 4 \times 7} \\ A &= \arccos(-0.625) \\ A &= 128.7^\circ \end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned} \int_1^4 x^2 + 4\ dx &= \left[\frac{x^3}{3} + 4x\right]_1^4 \\ &= \frac{64}{3} + 16 - \frac{1}{3} - 4 \\ &= 33 \end{aligned}$$

[3 marks]

Answer 6

Add the two equations to get $2x^2 = 80 \Leftrightarrow x = \pm\sqrt{40}$

$(\pm\sqrt{40}, \pm\sqrt{24})$ or $(\pm 2\sqrt{10}, \pm 2\sqrt{6})$

or $(\pm 6.32, \pm 4.90)$

[4 marks]

Answer 7

$$2\cos^2 x - 3\sin x = 3$$

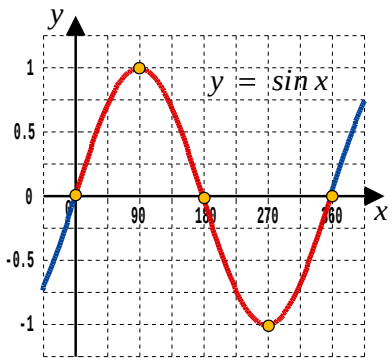
$$2(1 - \sin^2 x) - 3\sin x = 3 \quad \text{using } \cos^2 x + \sin^2 x = 1$$

$$2 - 2\sin^2 x - 3\sin x = 3$$

$$2\sin^2 x + 3\sin x + 1 = 0$$

$$(2\sin x + 1)(\sin x + 1) = 0$$

Either $\sin x = -\frac{1}{2}$ or $\sin x = -1$



$x_{WA} = 30°$

$x = \{210°, 270°, 330°\}$

[6 marks]

Answer 8

$$f^{-1}(x) = \frac{3x - 5}{4}$$

[3 marks]

Answer 9

(a)

$$s = ut + \frac{1}{2} a t^2$$

$$50 = 5 \times 4 + \frac{1}{2} \times a \times 4^2$$

$$a = 3.75 \text{ m s}^{-2}$$

[3 marks]

(b)

$$v^2 = u^2 + 2as$$

$$30^2 = 5^2 + 2 \times 3.75 \times s$$

$$s = 116.67 \text{ metres}$$

[3 marks]

Answer 10

$$\frac{dy}{dx} = 3x^2 \qquad \text{so at } \left(\sqrt{2},\ 2\sqrt{2}\right) \quad m = 6$$

$$y = mx + c \Rightarrow 2\sqrt{2} = 6\sqrt{2} + c \Rightarrow c = -4\sqrt{2}$$

$$y = 6x - 4\sqrt{2}$$

Additional Mathematics
Grade Gobbler 7

40 Mark Paper

Question 1

A cone's circular base, where x and y are in cm, is described by the equation

$$x^2 + y^2 + 6x - 14y = 6$$

(i) Write the equation of the circular base in the form

$$(x - a)^2 + (y - b)^2 = r^2$$

[2 marks]

(ii) The cone has a perpendicular height, h, of 13.5 cm.
Determine the volume of the cone, correct to 1 decimal place.

$$Voume\ of\ a\ cone = \frac{\pi}{3} r^2 h$$

[1 mark]

Question 2

By using Pascal's Triangle, or otherwise, expand the brackets;

$$\left(\frac{1}{3x} + x \right)^4$$

[3 marks]

Question 3

This question is about plotting the graph of the quadratic curve $y = x^2 + 4x - 5$ and then using the graph to solve an inequality.

(i) Rajit has decided to make a table to assist him in determining some points on the quadratic curve $y = x^2 + 4x - 5$

He has calculated that $(-6,\ 7)$ is on the curve.

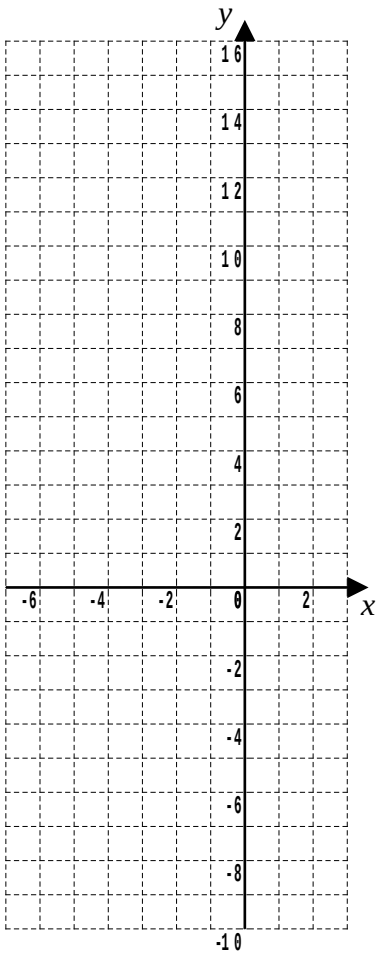
Help Rajit by completing his table.

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
x^2		36									
$4x - 5$		-29									
y		7									

[2 marks]

(ii) Use the table to plot the quadratic curve $y = x^2 + 4x - 5$

Place the equation of the curve next to the curve plotted.



[2 marks]

(iii) Factorise $x^2 + 4x - 5$

[1 mark]

(iv) There is a link between the graph of the curve $y = x^2 + 4x - 5$ and its factorisation.
Look at the curve carefully and try to spot the connection.
State the connection;

[1 mark]

(v) Explain the connection

[1 mark]

(vi) Solve the inequality $x^2 + 4x < 5$

[2 marks]

Question 4

Prove that $\frac{\sqrt{1 - cos^2\theta}}{\sqrt{1 - sin^2\theta}} = tan\,\theta$

[2 marks]

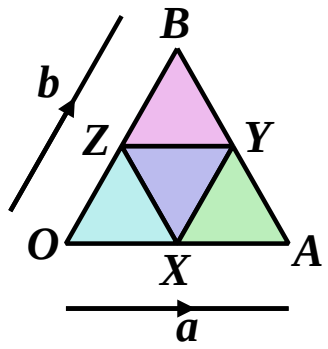
Question 5

(Regular GCSE Revision Question)

The diagram below shows four congruent equilateral triangles arranged edge-to-edge to form a large equilateral triangle.

Notice that; $\overrightarrow{OA} = \boldsymbol{a}$

and; $\overrightarrow{OB} = \boldsymbol{b}$



Describe, in terms of the vectors ***a*** and ***b***, the following;

(i) $\overrightarrow{OX}$ **(ii)** $\overrightarrow{AX}$ **(iii)** $\overrightarrow{ZB}$

(iv) $\overrightarrow{YZ}$ **(v)** $\overrightarrow{ZO}$ **(vi)** $\overrightarrow{BA}$

(vii) $\overrightarrow{ZX}$ **(viii)** $\overrightarrow{OY}$ **(ix)** $\overrightarrow{AZ}$

[3 marks]

Question 6

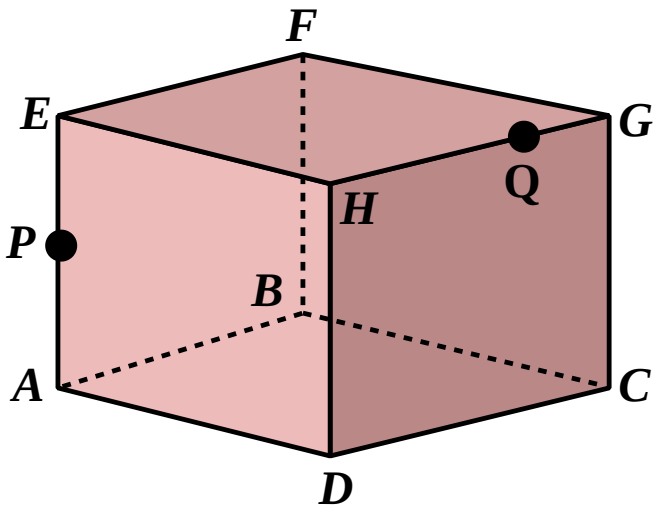
Differentiate, $y = \dfrac{4x^3 + 3x^5}{x^2}$

[2 marks]

Question 7

ABCDEFGH is a cube of side 4 cm.

AP = 2 cm and *HQ* = 3 cm.



Calculate;

(i) the lengths of *EQ* and *PD*

[2 marks]

(ii) the length of *PQ*

[1 mark]

(iii) the angle between *PQ* and the plane *EFGH*

[2 marks]

(iv) the angle *QPD*

[3 marks]

Question 8

(i) Show that 144 is a factor of 1296

[1 mark]

(ii) Consider the graph of the equation,

$$y = 36x^4 - 648x^2 + 18$$

Determine $\frac{dy}{dx}$

[1 mark]

(iii) Hence determine all three turning points.

[3 marks]

(iv) Determine $\frac{d^2y}{dx^2}$

[1 mark]

(v) Hence, for each turning point, determine if it is a local minimum or or a local maximum.

[2 marks]

Question 9

Determine $\int_0^1 (x + 5)(2x^2 + 1)\, dx$

[2 marks]

Grade Gobbler 7

Answer 1

(i)

$$x^2 + y^2 + 6x - 14y = 6$$

$$\left[x^2 + 6x \right] + \left[y^2 - 14y \right] = 6$$

$$\left[(x + 3)^2 - 9 \right] + \left[(y - 7)^2 - 49 \right] = 6$$

$$(x + 3)^2 + (y - 7)^2 = 8^2$$

[2 marks]

(ii)

$$V = \frac{\pi}{3} \times 64 \times 13.5$$

$$= 904.8 \text{ cm}^3$$

Answer 2

$$\left(\frac{1}{3x} + x \right)^4 = (1) \times \left(\frac{1}{3x} \right)^4 \times (x)^0 + (4) \times \left(\frac{1}{3x} \right)^3 \times (x)^1 + (6) \times \left(\frac{1}{3x} \right)^2 \times (x)^2 + (4) \times \left(\frac{1}{3x} \right)^1 \times (x)^3 + (1) \times \left(\frac{1}{3x} \right)^0 \times (x)^4$$

$$= \frac{1}{81x^4} + \frac{4}{27x^2} + \frac{2}{3} + \frac{4x^2}{3} + x^4$$

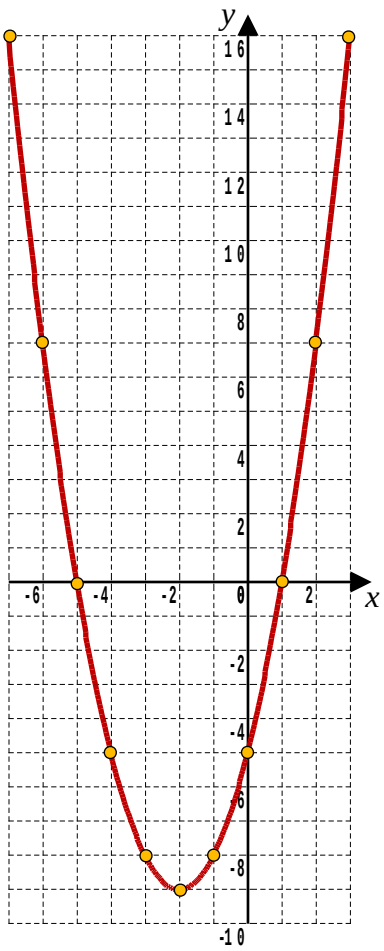
Answer 3

(i)

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
x^2	49	36	25	16	9	4	1	0	1	4	9
$4x - 5$	-33	-29	-25	-21	-17	-13	-9	-5	-1	3	7
y	16	7	0	-5	-8	-9	-8	-5	0	7	16

[2 marks]

(ii)



[2 marks]

(iii) $x^2 + 4x - 5 = (x + 5)(x - 1)$

[1 mark]

(iv) Crosses the x-axis when $x = -5$, $x = 1$

[1 mark]

(v) Crosses x-axis when $y = 0$.

When $(x + 5)(x - 1) = 0$ either $x = -5$ or $x = 1$

[1 mark]

(vi) $-5 < x < 1$

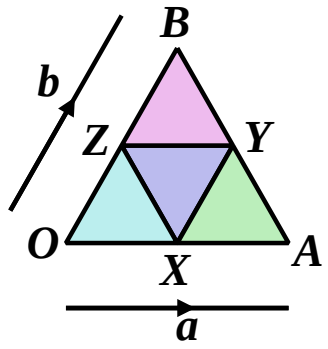
[2 marks]

Answer 4

$$\begin{aligned} \text{LHS} &= \frac{\sqrt{1 - \cos^2\theta}}{\sqrt{1 - \sin^2\theta}} \\ &= \frac{\sqrt{\sin^2\theta}}{\sqrt{\cos^2\theta}} \\ &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta \\ &= \text{RHS} \qquad \square \end{aligned}$$

[2 marks]

Answer 5



(i) $\overrightarrow{OX} = \frac{1}{2}\boldsymbol{a}$ **(ii)** $\overrightarrow{AX} = -\frac{1}{2}\boldsymbol{a}$ **(iii)** $\overrightarrow{ZB} = \frac{1}{2}\boldsymbol{b}$

(iv) $\overrightarrow{YZ} = -\frac{1}{2}\boldsymbol{a}$ **(v)** $\overrightarrow{ZO} = -\frac{1}{2}\boldsymbol{b}$ **(vi)** $\overrightarrow{BA} = \boldsymbol{a} - \boldsymbol{b}$

(vii) $\overrightarrow{ZX} = \frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

(viii) $\overrightarrow{OY} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

(ix) $\overrightarrow{AZ} = -\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

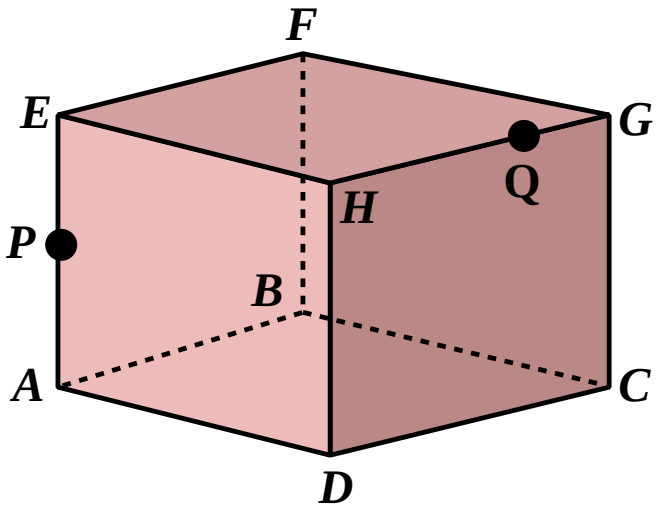
[3 marks]

Answer 6

$$\begin{aligned} y &= \frac{4x^3 + 3x^5}{x^2} \\ &= \frac{4x^3}{x^2} + \frac{3x^5}{x^2} \\ &= 4x + 3x^3 \\ \frac{dy}{dx} &= 4 + 9x^2 \end{aligned}$$

[2 marks]

Answer 7



(i) $|EQ| = \sqrt{4^2 + 3^2}$ $|PD| = \sqrt{2^2 + 4^2}$

$= 5$ cm $= 2\sqrt{5}$ cm (About 4.472 cm)

[2 marks]

(ii) $|PQ| = \sqrt{2^2 + 5^2}$

$= \sqrt{29}$ cm (About 5.385 cm)

[1 mark]

(iii) the angle between PQ and the plane $EFGH$ is $\angle PQE$

$$\angle PQE = \arctan\left(\frac{2}{\sqrt{29}}\right)$$

$$= 20.4°$$

[2 marks]

(iv) For angle QPD first use Pythagoras' Theorem, then the Cosine Rule backwards on triangle QPD

$$|QD| = \sqrt{4^2 + 3^2}$$

$$= 5 \text{ cm}$$

$$\cos P = \frac{q^2 + d^2 - p^2}{2qd}$$

$$= \frac{(2\sqrt{5})^2 + (\sqrt{29})^2 - 5^2}{2 \times 2\sqrt{5} \times \sqrt{29}}$$

$$= \frac{20 + 29 - 25}{4\sqrt{145}}$$

$$= 0.4983$$

$$P = 60.1°$$

[3 marks]

Answer 8

(i) $1296 = 9 \times 144 \quad \therefore$ 144 is a factor of 1296

[1 mark]

(ii) $$y = 36x^4 - 648x^2 + 18$$

$$\frac{dy}{dx} = 144x^3 - 1296x$$

$$= 144x(x^2 - 9)$$

[1 mark]

(iii) $\frac{dy}{dx} = 0$ when $x = 0$ or $x = \pm 3$

Turning Points : $(0,\ 18)$ $(\pm 3, -2898)$

[3 marks]

(iv) $$\frac{d^2y}{dx^2} = 432x^2 - 1296$$

[1 mark]

(v) When $x = 0$ $\frac{d^2y}{dx^2}$ is $-ve$ $\therefore$ $(0,\ 0)$ is a local minimum

When $x = \pm 3$ $\frac{d^2y}{dx^2}$ is $+ve$ $\therefore$ $(\pm 3, -2898)$ are local maxima

[2 marks]

Answer 9

$$\int_0^1 (x + 5)(2x^2 + 1)\ dx = \int_0^1 2x^3 + 10x^2 + x + 5\ dx$$

$$= \left[\frac{x^4}{2} + \frac{10x^3}{3} + \frac{x^2}{2} + 5x \right]_0^1$$

$$= 9\frac{1}{3}$$

[2 marks]

Additional Mathematics
Grade Gobbler 8

40 Mark Paper

Question 1

Solve the inequality

$$-13 \leqslant 5x + 7 \leqslant 42$$

[2 marks]

Question 2

(i) Differentiate;

$$y = x^3 + 4x^{-2}$$

[2 marks]

(ii) What is the value of the gradient on the curve $y = x^3 + 4x^{-2}$ at the point with coordinates (2, 9) ?

[2 marks]

(iii) Find the equation of the tangent to the curve $y = x^3 + 4x^{-2}$ at the point (2, 9).
Write your answer in the form $y = mx + c$

[2 marks]

Question 3

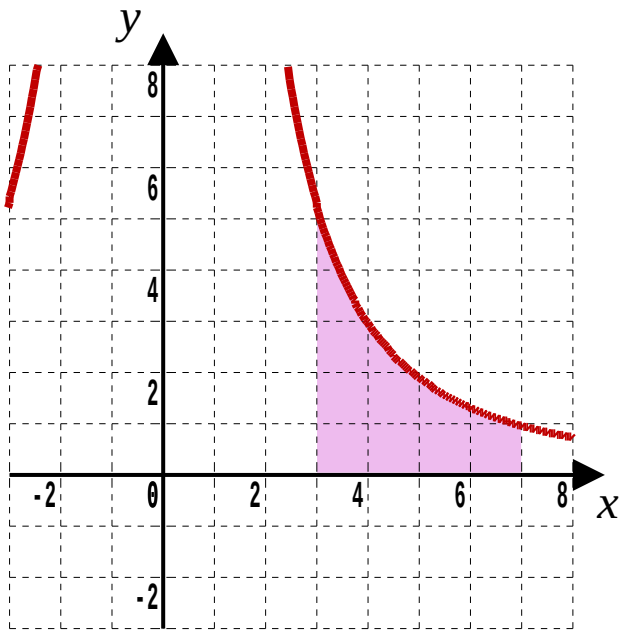
The diagram below shows, shaded, the area bounded by;

the x-axis

the line $x = 3$,

the line $x = 7$,

and the curve $y = \frac{48}{x^2}$



Use integration to find the area shaded.

[4 marks]

Question 4

The coordinates of two points A and B are $A\ (-2, -3)$ and $B\ (6, 9)$

(i) Write down the coordinates of C, the mid-point between A and B

[1 mark]

(ii) What is the gradient of the line through A and B ?

[1 mark]

(iii) What is the gradient of any line perpendicular to the line through A and B ?

[1 mark]

(iv) Determine the equation of the perpendicular bisector of the piece of line the lies between A and B.

[2 marks]

(v) Write down the equation of the circle with centre C that has line AB as diameter.

[3 marks]

Question 5

A coin is biased such that, when tossed, the probability of it landing heads is 0.7
This coin is tossed twelve times.
Use the formula;

$$P(\,r\ heads\,) \;=\; {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

to calculate the probability that exactly seven of the twelve tosses are heads.

[4 marks]

Question 6

Equations of the form, $a\,x^2 + bx + c = 0$, (where a, b and c are constants),
can be solved by means of the formula;

$$x \;=\; \frac{-b \;\pm\; \sqrt{b^2 - 4ac}}{2a}$$

Show how to use this formula to find the two values of x that satisfy the equation;

$$x^2 + 4x - 18 = 0$$

Write your answers in the form;

$$x \;=\; p \;\pm\; \sqrt{q}$$

where p and q are integers and p is a prime and q square free.

[3 marks]

Question 7

$$f(x) = x^2 - 11x + 30$$

(i) Where does the graph of $y = f(x)$ cross the x-axis ?

[2 marks]

(ii) Sketch the general shape of the graph of $y = f(x)$
On your rough sketch, mark the x-axis crossing points found in part (i).

[2 marks]

(iii) Solve the inequality $x^2 - 11x + 30 \leqslant 0$

[2 marks]

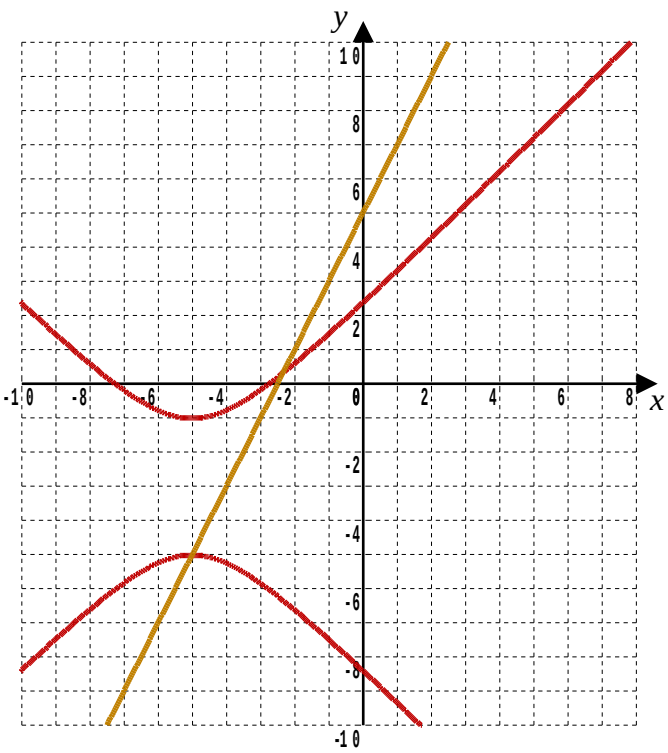
(iv) Solve the inequality $x^2 - 11x + 30 > 0$

[2 marks]

Question 8

On the graph below is plotted the straight line with equation, $y = 2x + 5$

and the hyperbola with equation, $(y + 3)^2 - (x + 5)^2 = 4$



Find the coordinates of the two points where the line and hyperbola intersect.
Give your answers accurate to 3 decimal places, where appropriate.

[5 marks]

Grade Gobbler 8

Answer 1

$$-13 \leqslant 5x + 7 \leqslant 42$$

$$-20 \leqslant 5x \leqslant 35$$

$$-4 \leqslant x \leqslant 7$$

[2 marks]

Answer 2

(i)

$$y = x^3 + 4x^{-2}$$

$$\frac{dy}{dx} = 3x^2 - 8x^{-3}$$

$$= 3x^2 - \frac{8}{x^3}$$

[2 marks]

(ii) When $x = 2$, $\frac{dy}{dx} = 11$

[2 marks]

(iii)

$$y = mx + c$$

$$9 = 11 \times 2 + c \qquad \therefore c = -13$$

$$y = 11x - 13$$

[2 marks]

Answer 3

$$\int_3^7 \frac{48}{x^2}\,dx = \int_3^7 48x^{-2}\,dx$$

$$= \left[-48x^{-1}\right]_3^7$$

$$= \left[-\frac{48}{x}\right]_3^7$$

$$= \left[\left(-\frac{48}{7}\right) - \left(-\frac{48}{3}\right)\right]$$

$$= \frac{64}{7} \quad \text{or} \quad 9.14$$

[4 marks]

Answer 4

(i) $$Midpoint = \left(\frac{-2 + 6}{2}, \frac{-3 + 9}{2} \right)$$

$$= (2, 3)$$

[1 mark]

(ii) $$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{9 - (-3)}{6 - (-2)}$$

$$= \frac{12}{8}$$

$$= \frac{3}{2}$$

[1 mark]

(iii) Perpendicular gradient: "sign changed reciprocal"

i.e. $-\frac{2}{3}$

[1 mark]

(iv) $$y = mx + c$$

$$3 = -\frac{2}{3} \times 2 + c \qquad \therefore \; c = \frac{13}{3}$$

$$y = -\frac{2}{3}x + \frac{13}{3}$$

[2 marks]

(v) $Radius = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ between B and the midpoint

$$= \sqrt{(6 - 2)^2 + (9 - 3)^2}$$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52} \quad \text{or} \quad 2\sqrt{13} \quad \text{or} \quad 7.211$$

Equation of circle;

$$(x - 2)^2 + (y - 3)^2 = 52$$

[3 marks]

Answer 5

$$P(7 \, heads) = {}^{12}C_7 \, (0.7)^7 (0.3)^5$$

$$= 0.158\,496$$

[4 marks]

Answer 6

You will get MO MARKS in this question if you use the "solve equation" on a calculator to get solutions without any working.

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-18)}}{2 \times 1}$$

$$x = \frac{-4 \pm \sqrt{88}}{2}$$

$$x = \frac{-4 \pm \sqrt{4 \times 22}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{22}}{2}$$

$$x = \frac{2\left(-2 \pm \sqrt{22}\right)}{2}$$

$$x = -2 \pm \sqrt{22}$$

[3 marks]

Answer 7

(i) $f(x) = 0 \Rightarrow x^2 - 11x + 30 = 0$

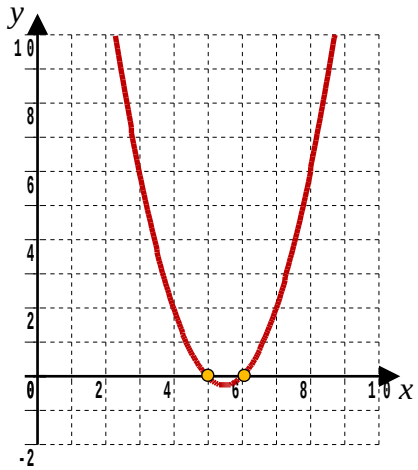
$(x - 5)(x - 6) = 0$

Either $(x - 5) = 0$ or $(x - 6) = 0$

$\therefore$ Crosses the x-axis at $(5, 0)$, $(6, 0)$

[2 marks]

(ii)



[2 marks]

(iii) $-5 \leqslant x \leqslant 6$

[2 marks]

(iv) $x < 5$ or $x > 6$

[2 marks]

Answer 8

$$(y+3)^2-(x+5)^2=4$$

$$(2x+5+3)^2-(x+5)^2=4$$

$$(2x+8)^2-(x+5)^2=4$$

$$4x^2+32x+64-x^2-10x-25=4$$

$$3x^2+22x+35=0$$

$$(3x+7)(x+5)=0$$

$$x=-\frac{7}{3} \quad \text{or} \quad -5$$

$$\left(-\frac{7}{3},\ \frac{1}{3}\right) \quad \text{or} \quad (-5,-5)$$

[5 marks]

Additional Mathematics
Grade Gobbler 9

40 Mark Paper

Question 1

$$y = \frac{x^2 + 4x^5}{x^3}$$

Find $\frac{dy}{dx}$

[3 marks]

Question 2

(i) Expand the brackets;

$$\left(2x + \frac{1}{x}\right)^4$$

[4 marks]

(ii) Which term is independent of x ?

[1 mark]

(iii) In your expansion what is the coefficient of x^{-2} ?

[1 mark]

Question 3

The diameter of a circle has endpoints (3, 5) and (51, 19)

(i) What is the radius of the circle ?

[2 marks]

(ii) What are the coordinates of the centre of the circle ?

[2 marks]

(iii) Write down the equation of the circle in the form;

$$(x + p)^2 + (y + q)^2 = k$$

where p, q and k are constants to be determined.

[2 marks]

Question 4

A standard, unbiased, six sided die is rolled twelve times.

(i) How many times, on average, would you expect a *four* to be rolled ?

[1 mark]

(ii) Use the formula;

$$P(r\,fours) = {}^{n}C_r\, p^r\, q^{n-r}$$

to calculate the probability that exactly three of the twelve rolls are *four*.

[3 marks]

Question 5

$$f(x) = x^2 - 2x - 15$$

(i) Where does the graph of $y = f(x)$ cross the x-axis ?

[1 mark]

(ii) Sketch the general shape of the graph of $y = f(x)$
On your rough sketch, mark the x-axis crossing points found in part (i)

[1 mark]

(iii) Solve the $x^2 - 2x - 15 < 0$

[2 marks]

(iv) Solve the inequality $x^2 - 2x - 15 \geqslant 0$

[2 marks]

Question 6

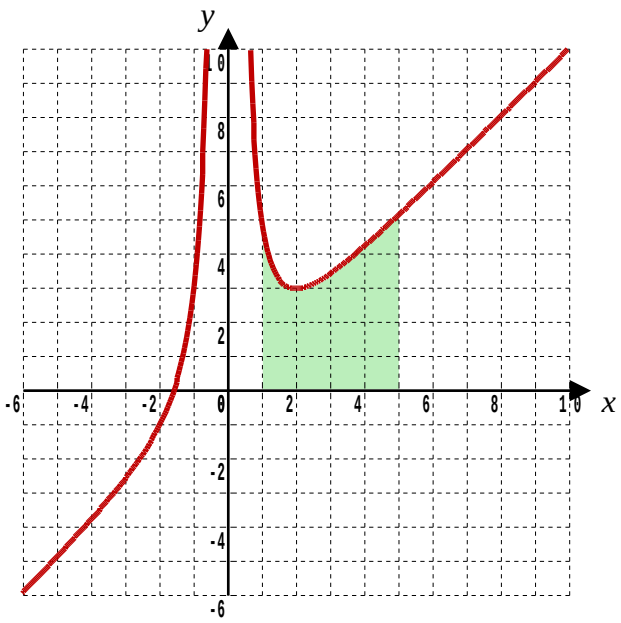
The diagram below shows, shaded, the area bounded by;

the x-axis

the line $x = 1$

the line $x = 5$

and the curve $y = x + \frac{4}{x^2}$



Use integration to find the exact area shaded.

[4 marks]

Question 7

(i) Write down the equation of the line perpendicular to the line with equation;

$$y = -\frac{1}{2}x + 6$$

and which passes through the point (5, 11)

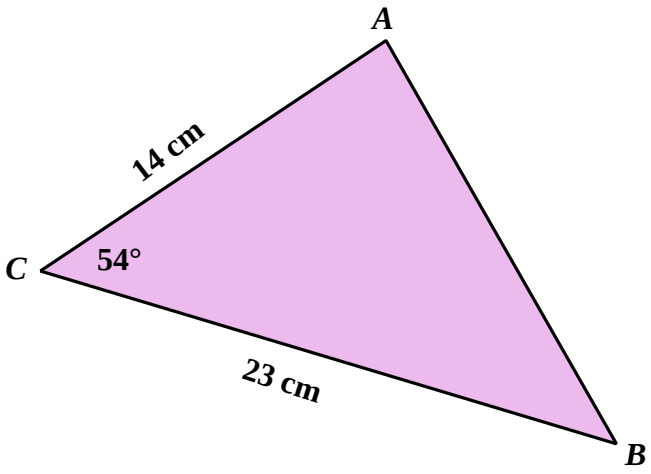
[2 marks]

(ii) What are the coordinates of the point where these two lines intersect ?

[2 marks]

Question 8

Calculate the length of *AB*



[3 marks]

Question 9

Find all of the turning points on the curve with equation;

$$y = \frac{1}{2}x^4 - 4x^2 + 3$$

clearly stating which are local minima and which are local maxima.

[4 marks]

Answers to Grade Gobbler 9

Answer 1

$$\begin{aligned} y &= \frac{x^2 + 4x^5}{x^3} \\ &= \frac{x^2}{x^3} + \frac{4x^5}{x^3} \\ &= x^{-1} + 4x^2 \\ \frac{dy}{dx} &= -x^{-2} + 8x \\ &= 8x - \frac{1}{x^2} \end{aligned}$$

[3 marks]

Answer 2

(i) $$\left(2x + \frac{1}{x}\right)^4 = (1) \times (2x)^4 \times \left(\frac{1}{x}\right)^0 + (4) \times (2x)^3 \times \left(\frac{1}{x}\right)^1 + (6) \times (2x)^2 \times \left(\frac{1}{x}\right)^2 + (4) \times (2x)^1 \times \left(\frac{1}{x}\right)^3 + (1) \times (2x)^0 \times \left(\frac{1}{x}\right)^4$$

$$= 16x^4 + 32x^2 + 24 + \frac{8}{x^2} + \frac{1}{x^4}$$

(ii) 24

[1 mark]

(iii) 8

[1 mark]

Answer 3

(i) $$Radius = \frac{1}{2} \times \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$= \frac{1}{2} \times \sqrt{(51 - 3)^2 + (19 - 5)^2}$$

$$= \frac{1}{2} \times \sqrt{48^2 + 14^2}$$

$$= 25$$

[2 marks]

(ii) $$Midpoint = \left(\frac{3 + 51}{2}, \frac{5 + 19}{2} \right)$$

$= (27,\ 12)$ which is the centre of the circle

[2 marks]

(iii) $$(x - 27)^2 + (y - 12)^2 = 625$$

i.e. $p = -27$, $q = -12$ and $k = 625$

[2 marks]

Answer 4

(i) twice

[1 mark]

(ii) $$P(3 fours) = {}^{12}C_3 \left(\frac{1}{6} \right)^3 \left(\frac{5}{6} \right)^9$$

$$= 0.197\,396$$

[3 marks]

Answer 5

(i) $f(x) = 0 \Rightarrow x^2 - 2x - 15 = 0$

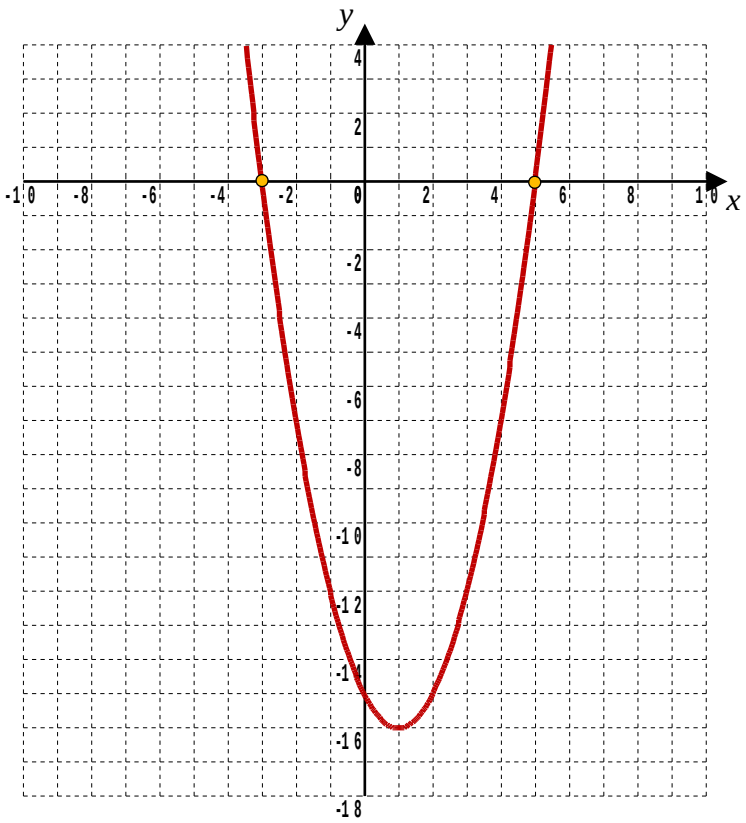
$(x + 3)(x - 5) = 0$

Either $(x + 3) = 0$ or $(x - 5) = 0$

$\therefore$ Crosses the x-axis at $(-3, 0)$, $(5, 0)$

[1 mark]

(ii)



[1 mark]

(iii) $-3 < x < 5$

[2 marks]

(iv) $x \leqslant -3$ or $x \geqslant 5$

[2 marks]

Answer 6

$$\int_1^5 x + \frac{4}{x^2}\, dx = \int_1^5 x + 4x^{-2}\, dx$$

$$= \left[\frac{x^2}{2} - 4x^{-1} \right]_1^5$$

$$= \left[\frac{x^2}{2} - \frac{4}{x} \right]_1^5$$

$$= \left(\frac{25}{2} - \frac{4}{5} \right) - \left(\frac{1}{2} - \frac{4}{1} \right)$$

$$= \frac{76}{5} \quad \text{or} \quad 15.2$$

[4 marks]

Answer 7

(i) $y = -\frac{1}{2}x + 6$ has a gradient of $-\frac{1}{2}$

Perpendicular gradient: "sign changed reciprocal" i.e. 2

$y = mx + c \Rightarrow 11 = 2 \times 5 + c \quad \therefore \; c = 1$

$y = 2x + 1$ is the required line

[2 marks]

(ii) $2x + 1 = -\frac{1}{2}x + 6$

$x = 2$ *so* $(2,\ 5)$ is the point of intersection

[2 marks]

Answer 8

By the cosine Rule,

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$= 23^2 + 14^2 - 2 \times 23 \times 14 \times \cos 54$$

$$= 346.466$$

$$c = 18.6 \text{ cm}$$

[3 marks]

Answer 9

$$y = \frac{1}{2}x^4 - 4x^2 + 3$$

$$\frac{dy}{dx} = 2x^3 - 8x$$

$$\frac{dy}{dx} = 0 \quad \text{when} \quad 2x^3 - 8x = 0$$

$$\text{i.e. when } 2x\left(x^2 - 4\right) = 0$$

$$2x\,(x-2)(x+2) = 0$$

$$\text{Either } x = 0 \quad \text{or} \quad x = \pm 2$$

$$(0,\ 3),\quad (\pm 2, -5)$$

$$\frac{d^2y}{dx^2} = 6x^2 - 8$$

$$(0,\ 3) \Rightarrow \frac{d^2y}{dx^2} \text{ is negative} \Rightarrow \text{Local MAXimum}$$

$$(\pm 2, -5) \Rightarrow \frac{d^2y}{dx^2} \text{ is positive} \Rightarrow \text{Local MINimum}$$

[4 marks]