

1.5 Answers

A-Level Pure Mathematics : Year 1 Differentiation II

1.5.1 Solutions to 1.2 Introductory Examples

$$(i) \quad y = \frac{7}{2}x^4 + \frac{1}{3}x^2 + 5x + 3 \quad \therefore \frac{dy}{dx} = 14x^3 + \frac{2}{3}x + 5$$

$$(ii) \quad y = 8\sqrt{x} + \frac{3}{x^2} \\ = 8x^{0.5} + 3x^{-2} \quad \therefore \frac{dy}{dx} = 4x^{-0.5} - 6x^{-3} \\ = \frac{4}{\sqrt{x}} - \frac{6}{x^3}$$

$$(iii) \quad y = \frac{4x + 7}{x} \\ = \frac{4x}{x} + \frac{7}{x} \\ = 4 + 7x^{-1} \quad \therefore \frac{dy}{dx} = -7x^{-2} \\ = -\frac{7}{x^2}$$

$$(iv) \quad y = (5 + \sqrt{x})^2 \\ = 25 + 10x^{0.5} + x \quad \therefore \frac{dy}{dx} = 5x^{-0.5} + 1 \\ = \frac{5}{\sqrt{x}} + 1$$

[8 marks]

1.5.2 Solutions (1.3 Exercise)

Answer 1

$$(i) \quad y = \frac{9}{5}x + \frac{4}{3}$$

$$\therefore \frac{dy}{dx} = \frac{9}{5}$$

$$(ii) \quad y = \frac{7x^4}{6}$$

$$\therefore \frac{dy}{dx} = \frac{14x^3}{3}$$

$$(iii) \quad y = (5x + 3)^2 \\ = 25x^2 + 30x + 9$$

$$\therefore \frac{dy}{dx} = 50x + 30$$

$$(iv) \quad y = 3 - 17x$$

$$\therefore \frac{dy}{dx} = -17$$

$$(v) \quad y = \frac{4}{x^{2.5}} \\ = 4x^{-2.5}$$

$$\therefore \frac{dy}{dx} = -10x^{-3.5} \\ = -\frac{10}{x^{3.5}}$$

$$(vi) \quad y = 18\sqrt{x} + \frac{5}{x^3} \\ = 18x^{0.5} + 5x^{-3}$$

$$\therefore \frac{dy}{dx} = 9x^{-0.5} - 15x^{-4} \\ = \frac{9}{\sqrt{x}} - \frac{15}{x^4}$$

[12 marks]

Answer 2

$$f(x) = \frac{5}{6}x^2 - \frac{7}{2}x^4$$

$$\therefore f'(x) = \frac{5}{3}x - 14x^3$$

[2 marks]

Answer 3

$$g(x) = \frac{4x^9}{3} + \frac{3x^6}{2}$$

$$\therefore g'(x) = 12x^8 + 9x^5$$

[2 marks]

Answer 4

$$h(x) = \frac{5}{4}x^2 + \frac{1}{3}x + 2$$

$$\therefore h'(x) = \frac{5}{2}x + \frac{1}{3}$$

[2 marks]**Answer 5**

$$p(x) = \frac{4 + x^2}{x^3}$$

$$= \frac{4}{x^3} + \frac{x^2}{x^3}$$

$$= 4x^{-3} + x^{-1}$$

$$\therefore p'(x) = -12x^{-4} - x^{-2}$$

$$= -\frac{12}{x^4} - \frac{1}{x^2}$$

[3 marks]**Answer 6**

$$y = \frac{10x + 1}{x^2}$$

$$= \frac{10x}{x^2} + \frac{1}{x^2}$$

$$= 10x^{-1} + x^{-2}$$

$$\therefore \frac{dy}{dx} = -10x^{-2} - 2x^{-3}$$

$$= -\frac{10}{x^2} - \frac{2}{x^3}$$

[3 marks]**Answer 7**

$$y = \frac{3x^2 + x}{\sqrt{x}}$$

$$= \frac{3x^2}{\sqrt{x}} + \frac{x}{\sqrt{x}}$$

$$= 3x^{1.5} + x^{0.5}$$

$$\therefore \frac{dy}{dx} = 4.5x^{0.5} + 0.5x^{-0.5}$$

$$= 4.5\sqrt{x} + \frac{0.5}{\sqrt{x}}$$

[3 marks]

Answer 8

With $n = 4$: $y = (3 - 2x)^4$

$$y = 81 - 216x + 216x^2 - 96x^3 + 16x^4$$

$$\therefore \frac{dy}{dx} = -216 + 432x - 288x^2 + 64x^3$$

With $n = 2$: $y = (3 - 2x)^2$

$$= 9 - 12x + 4x^2$$

$$\therefore \frac{dy}{dx} = -12 + 8x$$

[4 marks]**Answer 9**

$$w(x) = 3x^3 + \frac{2}{x^3}$$

$$= 3x^3 + 2x^{-3} \quad \therefore w'(x) = 9x^2 - 6x^{-4}$$

$$= 9x^2 - \frac{6}{x^4}$$

[3 marks]**Answer 10**

$$k(x) = \frac{2}{5x^4}$$

$$= \frac{2x^{-4}}{5} \quad \therefore k'(x) = -\frac{8x^{-5}}{5}$$

$$= -\frac{8}{5x^5}$$

[3 marks]**Answer 11**

$$m(x) = 3x^3 (2x^2 - x^5)$$

$$= 6x^5 - 3x^8 \quad \therefore m'(x) = 30x^4 - 24x^7$$

[3 marks]