

2.3 Answers

A-Level Pure Mathematics : Year 1 Differentiation II

2.3.1 Solution to Example 2.1

$$y = \frac{x^3}{9} - \frac{2x}{3}$$

$$\frac{dy}{dx} = \frac{x^2}{3} - \frac{2}{3}$$

$$\therefore \text{When } x = 3, \quad \frac{dy}{dx} = \frac{7}{3}$$

For the tangent, a line is required through (3, 1) with a gradient of $\frac{7}{3}$

$$y = mx + c$$

$$1 = \frac{7}{3} \times 3 + c \quad \text{gives} \quad c = -6$$

$$\text{So the tangent is, } y = \frac{7}{3}x - 6$$

Observe, on the graph, the tangent passes through (- 6, 0) and that from that point three across and seven up moves to (3, 1) so this result is visually verifiable.

For the normal, a line is required through (3, 1) with a gradient of $-\frac{3}{7}$

$$y = mx + c$$

$$1 = -\frac{3}{7} \times 3 + c \quad \text{gives} \quad c = \frac{16}{7}$$

$$\text{So the normal is, } y = -\frac{3}{7}x + \frac{16}{7}$$

Pleasingly, from an educational point of view, having been able to visually check the answer for the tangent, to get this exact answer the maths is needed, rather than simply looking at the graph of the Normal.

[8 marks]

2.3.2 Solutions to Exercise 2.2

Answer 1

(i) $\frac{dy}{dx} = \frac{12}{x^2}$

[2 marks]

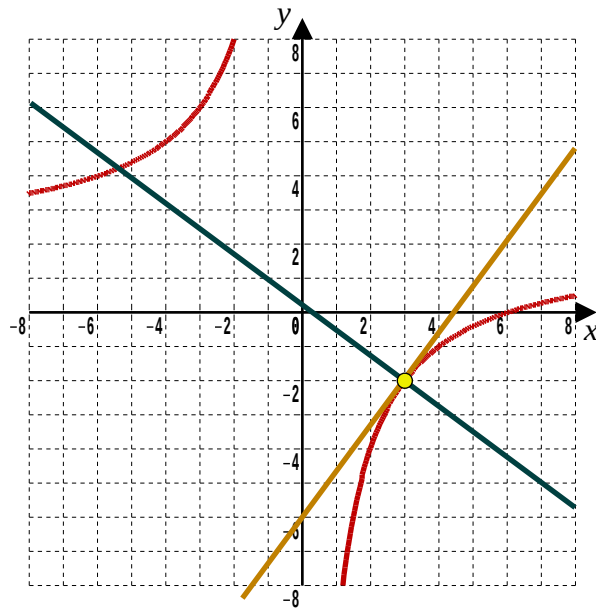
(ii) Tangent: $y = \frac{4}{3}x - 6$

[3 marks]

(iii) Normal: $y = -\frac{3}{4}x + \frac{1}{4}$

[3 marks]

(iv)



[2 marks]

Answer 2

(i) $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - 2$ or $\frac{dy}{dx} = \frac{3}{2}\sqrt{x} - 2$

[2 marks]

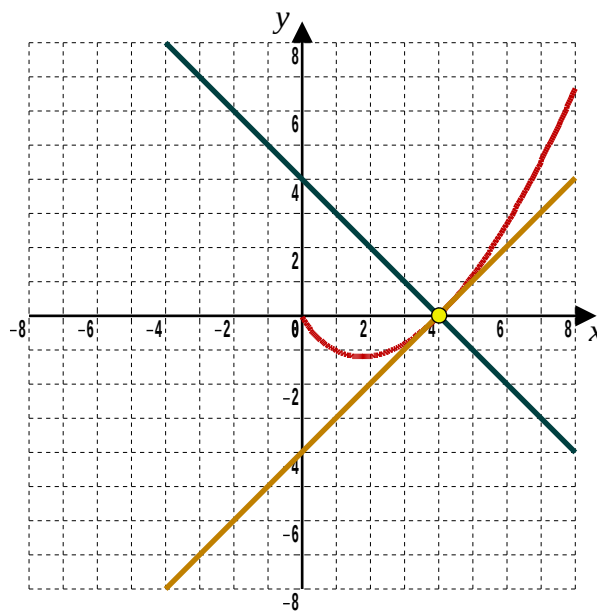
(ii) Tangent: $y = x - 4$

[3 marks]

(iii) Normal: $y = -x + 4$

[3 marks]

(iv)



[2 marks]

Answer 3

(i) $\frac{dy}{dx} = \frac{x}{8} - \frac{32}{x^3} - 2$

[2 marks]

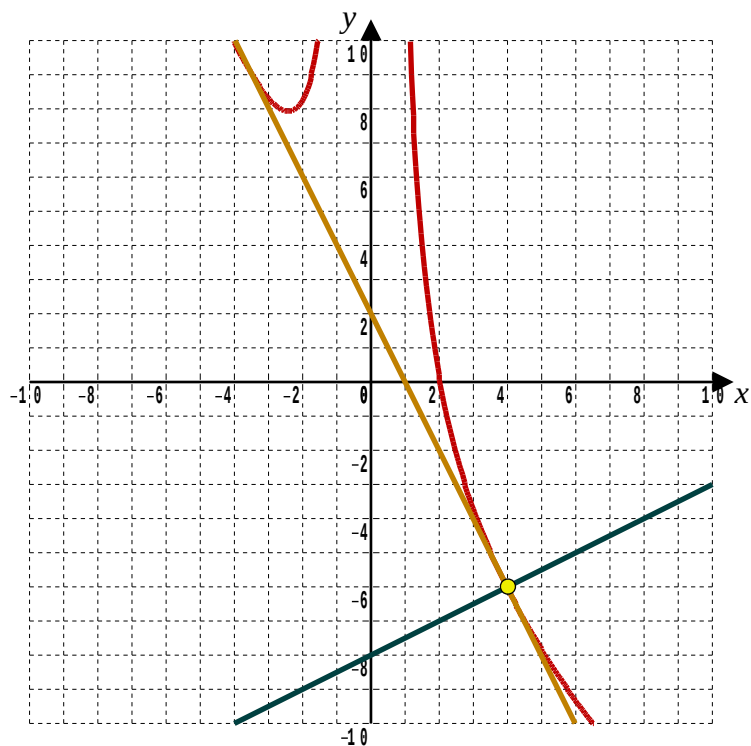
(ii) Tangent: $y = -2x + 2$

[3 marks]

(iii) Normal: $y = 0.5x - 8$

[3 marks]

(iv)



[3 marks]

Answer 4

(i) When $x = 2$, $y = \frac{2^4}{32} \left(\frac{2-5}{5} \right)$
 $= \frac{1}{2} \times \frac{-3}{5}$
 $= -\frac{3}{10}$
 $\therefore a = -0.3$

[2 marks]

(ii) $y = \frac{x^4}{32} \left(\frac{x-5}{5} \right)$
 $= \frac{x^5}{160} - \frac{x^4}{32}$
 $\frac{dy}{dx} = \frac{x^4}{32} - \frac{x^3}{8}$

[2 marks]

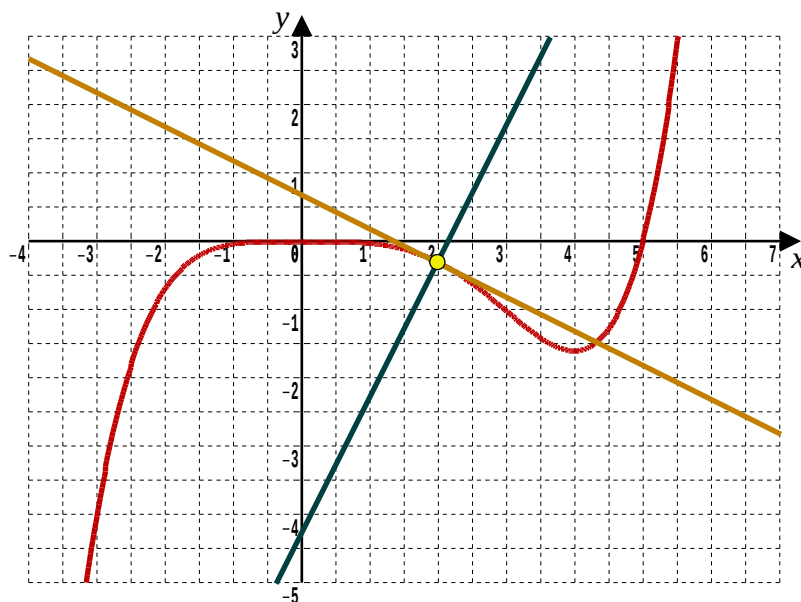
(iii) Tangent: $y = -0.5x + 0.7$

[3 marks]

(iv) Normal: $y = 2x - 4.3$

[3 marks]

(v)

**[2 marks]**

Answer 5

(i) $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}$

[2 marks]

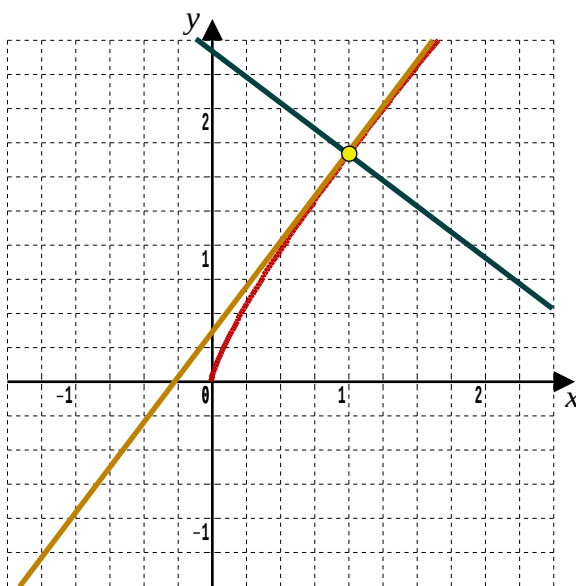
(ii) Tangent: $y = \frac{4}{3}x + \frac{1}{3}$

[3 marks]

(iii) Normal: $y = -\frac{3}{4}x + \frac{29}{12}$

[3 marks]

(iv)



[2 marks]