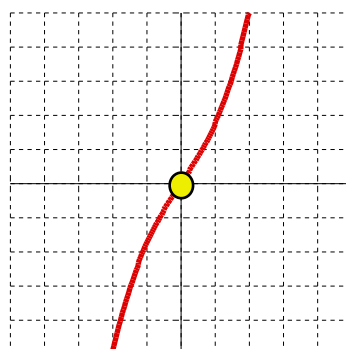
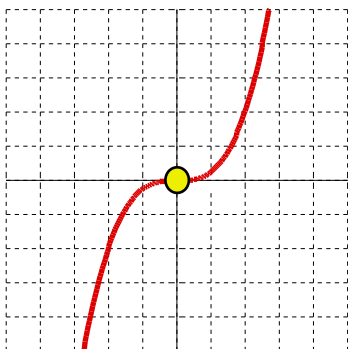


## 4.6 Answers

### A-Level Pure Mathematics : Year 1 Differentiation II

#### 4.6.1 Solution to 4.3 Example



Left : An example of a stationary point of inflection

This function is increasing

(Note that increasing can include points with gradient zero)

Right : An example of a non-stationary point of inflection

This function is strictly increasing

(Note that the “strictly” means there are no points with gradient zero)

[ 2 marks ]

#### 4.6.1 Solution to Example 4.4

$$(i) \quad y = x^3 + 6x^2 + 9x \quad \therefore \quad \frac{dy}{dx} = 3x^2 + 12x + 9$$

[ 2 marks ]

$$(ii) \quad \frac{d^2y}{dx^2} = 6x + 12$$

[ 2 marks ]

$$(iii) \quad \text{For a turning point, } 3x^2 + 12x + 9 = 0$$

$$x^2 + 4x + 3 = 0$$

$$(x + 3)(x + 1) = 0$$

$$\text{Either } x = -3 \text{ or } x = -1$$

$$\begin{aligned} y &= (-3)^3 + 6(-3)^2 + 9(-3) \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= (-1)^3 + 6(-1)^2 + 9(-1) \\ &= -4 \end{aligned}$$

The turning points are  $(-3, 0)$ ,  $(-1, -4)$

$$\text{For } x = -3, \quad \frac{d^2y}{dx^2} = -6$$

$$\text{For } x = -1, \quad \frac{d^2y}{dx^2} = +6$$

$(-3, 0)$  is a local maximum

$(-1, -4)$  is a local minimum

[ 4 marks ]

#### 4.6.2 Solutions to 4.5 Exercise

##### Answer 1

( i )  $y = 5x^2 - 30x \quad \therefore \quad \frac{dy}{dx} = 10x - 30$

[ 1 mark ]

( ii )  $\frac{d^2y}{dx^2} = 10$

[ 1 mark ]

( iii ) For a turning point,  $10x - 30 = 0 \quad \Leftrightarrow x = 3$

To get a **point** substitute this value of  $x$  back into the point equation of the curve. (NOT the gradient equation of the curve !)

$$\begin{aligned} y &= 5x^2 - 30x \\ &= 5 \times 3^2 - 30 \times 3 \\ &= -45 \Leftrightarrow (3, -45) \end{aligned}$$

When  $x = 3$ ,  $\frac{d^2y}{dx^2} = 10$

As this is positive  $(3, -45)$  is a local minimum.

[ 3 marks ]

##### Answer 2

( i )  $y = 4x^2 + 16x + 21 \quad \therefore \quad \frac{dy}{dx} = 8x + 16$

[ 2 marks ]

( ii ) For a turning point,  $8x + 16 = 0 \quad \Leftrightarrow x = -2$

To get a **point** substitute this value of  $x$  back into the point equation of the curve. (NOT the gradient equation of the curve !)

$$\begin{aligned} y &= 4x^2 + 16x + 21 \\ &= 4 \times (-2)^2 + 16 \times (-2) + 21 \\ &= 16 - 32 + 21 \\ &= 5 \Leftrightarrow (-2, 5) \end{aligned}$$

[ 4 marks ]

( iii ) Regardless of the value of  $x$ ,  $\frac{d^2y}{dx^2} = 8$

As this is positive  $(-2, 5)$  is a local minimum.

As this is positive is a local minimum.

[ 2 marks ]

**Answer 3**

$$(i) \quad y = x^3 + 9x^2 + 15x \quad \therefore \quad \frac{dy}{dx} = 3x^2 + 18x + 15$$

**[ 2 marks ]**

$$(ii) \quad \frac{d^2y}{dx^2} = 6x + 18$$

**[ 2 marks ]**

$$(iii) \quad \text{For a turning point, } 3x^2 + 18x + 15 = 0$$

$$x^2 + 6x + 5 = 0$$

$$(x + 5)(x + 1) = 0$$

$$\text{Either } x = -5 \text{ or } x = -1$$

$$y = x^3 + 9x^2 + 15x$$

$$= (-5)^3 + 9(-5)^2 + 15(-5)$$

$$= -125 + 225 - 75$$

$$= 25 \Leftrightarrow (-5, 25)$$

$$y = x^3 + 9x^2 + 15x$$

$$= (-1)^3 + 9(-1)^2 + 15(-1)$$

$$= -1 + 9 - 15$$

$$= -7 \Leftrightarrow (-1, -7)$$

$$\text{When } x = -5, \quad \frac{d^2y}{dx^2} = -12$$

$(-5, 25)$  is a local maximum

$$\text{When } x = -1, \quad \frac{d^2y}{dx^2} = +12$$

$(-1, -7)$  is a local minimum

**[ 4 marks ]****Answer 4**

$$(i) \quad f(x) = x^3 + 6x^2 + 5 \quad \therefore \quad f'(x) = 3x^2 + 12x$$

**[ 2 marks ]**

$$(ii) \quad f''(x) = 6x + 12$$

**[ 2 marks ]**

$$(iii) \quad \text{For a turning point, } 3x^2 + 12x = 0$$

$$x^2 + 4x = 0$$

$$x(x + 4) = 0$$

$$\text{Either } x = 0 \text{ or } x = -4$$

$$y = x^3 + 6x^2 + 5$$

$$= (0)^3 + 6(0)^2 + 5$$

$$= 5 \Leftrightarrow (0, 5)$$

$$y = x^3 + 6x^2 + 5$$

$$= (-4)^3 + 6(-4)^2 + 5$$

$$= 37 \Leftrightarrow (-4, 37)$$

$$\text{When } x = 0, \quad \frac{d^2y}{dx^2} = 12$$

$(0, 5)$  is a local minimum

$$\text{When } x = -4, \quad \frac{d^2y}{dx^2} = -12$$

$(-4, 37)$  is a local maximum

**[ 4 marks ]**

**Answer 5**

$$y = \sqrt{2} x^2 - 6\sqrt{x} + 4\sqrt{2}$$

$$= \sqrt{2} x^2 - 6x^{\frac{1}{2}} + 4\sqrt{2}$$

$$\frac{dy}{dx} = 2\sqrt{2} x - 3x^{-\frac{1}{2}}$$

$$= 2\sqrt{2} x - \frac{3}{\sqrt{x}}$$

$$\text{When } x = 2, \frac{dy}{dx} = 4\sqrt{2} - \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 4\sqrt{2} - \frac{3}{2}\sqrt{2}$$

$$= \frac{5}{2}\sqrt{2} \quad \therefore a = \frac{5}{2}$$

**[ 4 marks ]****Answer 6**

Given that students do not yet know of “The Product Rule”...

$$f(x) = (x - 2)^2 (2x + 1)$$

$$= (x^2 - 4x + 4)(2x + 1)$$

$$= 2x^3 - 8x^2 + 8x + x^2 - 4x + 4$$

$$= 2x^3 - 7x^2 + 4x + 4$$

$$f'(x) = 6x^2 - 14x + 4$$

$$\text{For turning points, } 6x^2 - 14x + 4 = 0$$

$$3x^2 - 7x + 2 = 0$$

$$(3x - 1)(x - 2) = 0$$

$$\text{Either } x = \frac{1}{3} \text{ or } x = 2$$

The minimum at (2, 0) is already known about.

$$f\left(\frac{1}{3}\right) = \left(\frac{1}{3} - 2\right)^2 \left(\frac{2}{3} + 1\right)$$

$$= \left(\frac{5}{3}\right)^2 \times \frac{5}{3}$$

$$= \frac{125}{27} \quad \therefore P \text{ is } \left(\frac{1}{3}, \frac{125}{27}\right)$$

**[ 7 marks ]**

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In October 2020, Shrewsbury School was voted “**Independent School of the Year 2020**”

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk