5.4 Answers

5.4.1 Solution to Example 5.2

(i)

$$y = x^3 + 12x$$

$$\frac{dy}{dx} = 3x^2 + 12$$

To find turning points,

$$\frac{dy}{dx} = 0 \quad \text{when} \quad 3x^2 + 12 = 0$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

This has no real solutions; the curve has no turning points. □

[3 marks]

This is an example of a strictly increasing function,.
Its gradient is always positive.

[1 mark]

5.4.2 Solution to Exercise 5.3

Answer 1

$$y = x^3 + 3x^2 - 24x$$

$$\frac{dy}{dx} = 3x^2 + 6x - 24$$

$$\frac{dy}{dx} = 0 \quad \text{when} \quad 3x^2 + 6x - 24 = 0$$

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

Either $x = -4$ or $x = 2$

The turning points are $(-4,\ 80)$, $(2, -28)$

$$\frac{d^2y}{dx^2} = 6x + 6$$

For $x = -4$, $\frac{d^2y}{dx^2} = -18$, so $(-4,\ 80)$ is a MAXimum

For $x = 2$, $\frac{d^2y}{dy^2} = 18$, so $(2, -28)$ is a MINimum

[5 marks]

Answer 2

$$y = 8x^2 + \frac{2}{x}$$

$$= 8x^2 + 2x^{-1}$$

$$\frac{dy}{dx} = 16x - 2x^{-2}$$

$$= 16x - \frac{2}{x^2}$$

$$\frac{dy}{dx} = 0 \quad \text{when} \quad 16x - \frac{2}{x^2} = 0$$

$$16x^3 - 2 = 0$$

$$8x^3 - 1 = 0$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

The turning point is $(0.5, 6)$

$$\frac{d^2y}{dx^2} = 16 + 4x^{-3}$$

$$= 16 + \frac{4}{x^3}$$

For $x = 0.5$, $\frac{d^2y}{dx^2} = 48$, so $(0.5, 6)$ is a MINimum

[5 marks]

Answer 3

(i) $\frac{dy}{dx} = 25x^4 - 21x^2$

[2 marks]

(ii) $\frac{d^2y}{dx^2} = 100x^3 - 42x$

[1 mark]

(iii) $(1, -2)$

[1 mark]

(iv) 4

[1 mark]

(v) 58

That is, positive bend $\therefore$ Anti-Clockwise

[2 marks]

Answer 4

(i) Consider perimeter: $2x + 2y = 72$

$$x + y = 36$$

$$\therefore\ y = 36 - x$$

Consider area: $A = x \times y$

$$= x\,(36 - x)$$

$$= 36x - x^2$$

[3 marks]

(ii) $36 - 2x$

[2 marks]

(iii) $x = 18$ cm $\Rightarrow$ $A = 324$ cm^2

[3 marks]

(iv) $\dfrac{d^2A}{dx^2} = -2$ As this is negative, the turning point is a MAXimum

[2 marks]

Answer 5

(a) $\dfrac{dy}{dx} = 5000 - 1250x$

[2 marks]

(b) (4, 10 000)

[3 marks]

(c) **(i)** Maximum

(ii) The curve is an upside down quadratic equation, $\cap$,
because of the negative in front of the x^2
and so only has a local maximum

[2 marks]

(d) **(i)** £4

(ii) It maximises his profit

[2 marks]

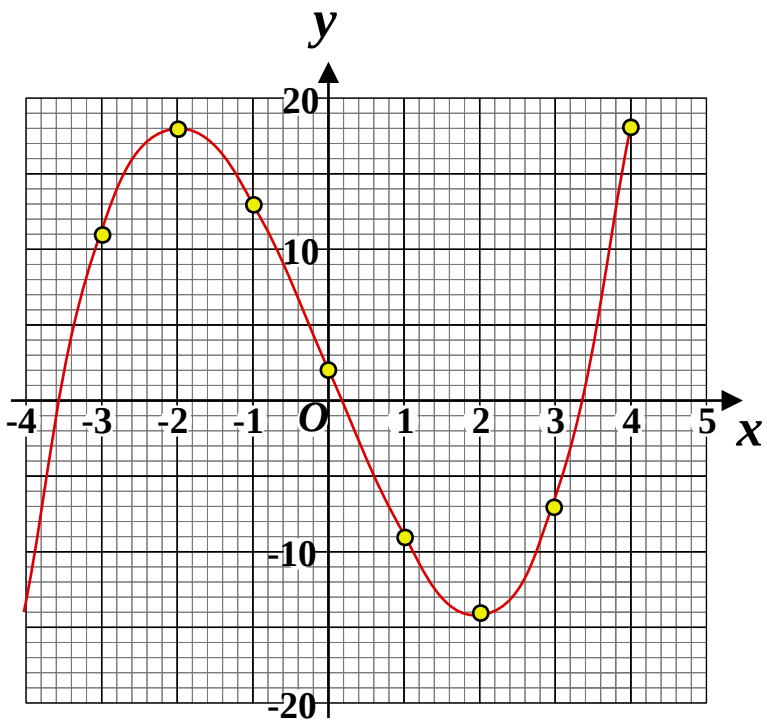
Answer 6

(a)

x	– 3	– 2	– 1	0	1	2	3	4
y	11	**18**	**13**	**2**	**– 9**	**– 14**	– 7	18

[2 marks]

(b)



[2 marks]

(c) **(i)** $\dfrac{dy}{dx} = 3t^2 - 12$

(ii) 63

[4 marks]

Answer 7

(a)

$$A = \text{area rectangle} - \text{area 4 triangles}$$

$$= 10 \times 8 - \frac{1}{2}(10 - x) \times 2 - \frac{1}{2}(8 - x)(x) \times 2$$

$$= 80 - x(10 - x) - x(8 - x)$$

$$= 80 - 10x + x^2 - 8x + x^2$$

$$= 2x^2 - 18x + 80$$

[3 marks]

(b) **(i)** $4x - 18$

(ii) 4.5

(iii) Second derivative is 4 which is positive
$\therefore$ Anticlockwise bend $\therefore$ MINimum

[5 marks]