### 6.5 Answers



### 6.5.1 Solution to Example 6.3

$$f(x) = \frac{x^5}{5} - 2x^3 + 13x$$

$$f'(x) = x^4 - 6x^2 + 13$$

This is a quadratic as can be made clear by letting $z = x^2$

$$\begin{aligned} f'(z) &= \left[ z^2 - 6z \right] + 13 \\ &= \left[ (z-3)^2 - 9 \right] + 13 \\ &= (z-3)^2 + 4 \end{aligned}$$

As the square has least value 0 and, even then 4 is added on 4 this expression is always positive

$\therefore\ f(x)$ is a strictly increasing function ☐

**[ 3 marks ]**

### 6.5.2 Solutions to Exercise 6.4

**Answer 1**

$$g(x) = x^4 + 32x + 50$$

$$g'(x) = 4x^3 + 32$$

$$g'(x) = 0 \quad \text{when} \quad 4x^3 + 32 = 0$$

$$x^3 = -8$$

$$x = -2$$

$$\begin{aligned} g(-2) &= 16 - 64 + 50 \\ &= 2 \end{aligned}$$

$$g''(x) = 12x^2$$

$g''(-2) = 48$ As this is positive, $(-2,\ 2)$ is a MINimum

**[ 3 marks ]**

**Answer 2**

$$y = 4x^5 + 8\sqrt{x} \qquad x \geqslant 0$$

$$= 4x^5 + 8x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 20x^4 + 4x^{-\frac{1}{2}}$$

$$= 20x^4 + \frac{4}{\sqrt{x}}$$

**[ 3 marks ]**

**Answer 3**

$$y = \frac{8}{x} - x + 3x^2 \qquad x > 0$$

$$= 8x^{-1} - x + 3x^2$$

When $x = 2$ $\quad y = 4 - 2 + 12$

$$= 14$$

$$\frac{dy}{dx} = -8x^{-2} - 1 + 6x$$

$$= -\frac{8}{x^2} - 1 + 6x$$

When $x = 2$, $\quad \frac{dy}{dx} = -2 - 1 + 12$

$$= 9 \quad \text{at} \quad (2,\ 14)$$

Tangent: $y = mx + c$

$$14 = 9 \times 2 + c$$

$$c = -4$$

The equation of the tangent is $y = 9x - 4$

**[ 4 marks ]**

**Answer 4**

**( i )** $\quad (x + h)^2 = x^2 + 2xh + h^2$

**[ 1 mark ]**

**( ii )** $\quad f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{5(x + h)^2 - 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{5\left(x^2 + 2xh + h^2\right) - 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 - 5x^2}{h}$$

$$= \lim_{h \to 0} \frac{10xh + 5h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(10x + 5h)}{h}$$

$$= \lim_{h \to 0} 10x + 5h$$

$$= 10x \qquad \square$$

**[ 4 marks ]**

**Answer 5**

**( i )** The 80 m of fencing has to run along two sides of length $y$ and one side of length $x$.

In other words, $80 = 2y + x \quad \Leftrightarrow \quad x = 80 - 2y$

The area enclosed is;

$$\begin{aligned} A &= yx \\ &= y\,(80 - 2y) \qquad \square \end{aligned}$$

**[ 2 marks ]**

**( ii )**

$$A = 80y - 2y^2$$

$$\frac{dA}{dy} = 80 - 4y$$

For a turning point $\dfrac{dA}{dy} = 0$

$$\begin{aligned} \therefore \quad 80 - 4y &= 0 \\ 4y &= 80 \\ y &= 20 \end{aligned}$$

The enclosure has dimensions 20 m by 40 m and area 800 $m^2$

**[ 3 marks ]**

**Answer 6**

$$\begin{aligned} y &= \frac{2x^3 + 5x}{x^3} \\ &= \frac{2x^3}{x^3} + \frac{5x}{x^3} \\ &= 2 + 5x^{-2} \\ \frac{dy}{dx} &= -10x^{-3} \\ &= -\frac{10}{x^3} \end{aligned}$$

**[ 3 marks ]**

**Answer 7**

$$y = 4x + 3x^{\frac{3}{2}} - 2x^2 \qquad x > 0$$

**( i )** $$\frac{dy}{dx} = 4 + \frac{9}{2}x^{\frac{1}{2}} - 4x$$

**[ 3 marks ]**

**( ii )** When $x = 4$,

$$y = 4 \times 4 + 3 \times 4^{\frac{3}{2}} - 2 \times 4^2$$
$$= 16 + 24 - 32$$
$$= 8$$

$\therefore$ ( 4, 8 ) is on the curve $C$ ☐

**[ 2 marks ]**

**( iii )** When $x = 4$, $$\frac{dy}{dx} = 4 + \frac{9}{2}\sqrt{4} - 4 \times 4$$
$$= 4 + 9 - 16$$
$$= -3$$

Thus $m_N = \frac{1}{3}$

Normal: $y = mx + c$

$$8 = \frac{1}{3} \times 4 + c$$
$$c = \frac{24 - 4}{3}$$
$$= \frac{20}{3}$$

The equation of the normal is thus $y = \frac{1}{3}x + \frac{20}{3}$

That is $3y = x + 20$ ☐

**[ 3 marks ]**

**( iv )** The normal will cut the $x$-axis when $y = 0$, when $x = -20$
Required, is the distance, $d$, between $(-20, 0)$ and $(4, 8)$

$$d = \sqrt{(\Delta y)^2 + (\Delta x)^2}$$
$$= \sqrt{8^2 + 24^2}$$
$$= 8\sqrt{10}$$

**[ 3 marks ]**