

Lesson 7

A-Level Pure Mathematics : Year 1 Differentiation II

7.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Total Marks Available : 53

Question 1

$$g(x) = x^4 - 256x + 800$$

Use calculus to find the stationary point on the graph of g and determine if it is a local maximum, a local minimum or a point of inflection.

[5 marks]

Question 2

A function has equation,

$$y = 4x^{1.5} + \frac{9}{x^3} \quad x \neq 0$$

Find the gradient equation of this curve.

[3 marks]

Question 3

A curve has equation

$$y = \sqrt{x} \quad x > 0$$

Find the equation of the tangent to the curve at the point where $x = 4$

[5 marks]

Question 4

$$f(x) = \frac{x^3}{3} - 3x^2 + 11x$$

Prove that $f(x)$ is a strictly increasing function.

[4 marks]

Question 5

(i) Expand the brackets;

$$(x + h)^3$$

[2 marks]

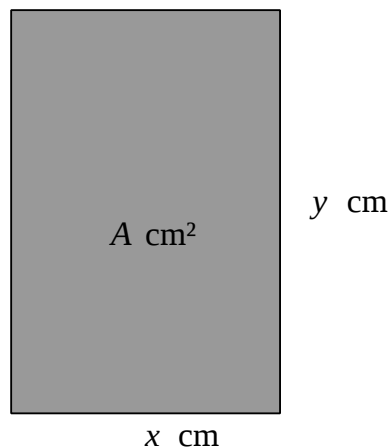
(ii) A derivative of a smooth and continuous function $f(x)$ is defined to be;

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

Prove, from first principles, that the derivative of $4x^3$ is $12x^2$

[4 marks]

Question 6



The diagram shows a rectangular photo frame of area $A \text{ cm}^2$.

The width of the photo frame is $x \text{ cm}$.

The height of the photo frame is $y \text{ cm}$.

The perimeter of the photo frame is 84 cm.

(i) Show that $A = 42x - x^2$

[3 marks]

(ii) Find $\frac{dA}{dx}$

[2 marks]

(iii) Find the maximum value of A .

[1 mark]

(iv) Show that $\frac{d^2A}{dx^2}$ detects that your part (iii) answer is a maximum.

[2 marks]

Question 7

The curve C has equation, $y = 17x + 3x^{\frac{2}{3}} - x^2 - 80 \quad x > 0$

- (i) Find an expression for $\frac{dy}{dx}$

[3 marks]

- (ii) Show that the point $P(8, 4)$ lies on C

[2 marks]

- (iii) Show that an equation of the normal to C at point P is $2y + x = 16$

[4 marks]

The normal to C at P cuts the x -axis at point Q

- (iv) Find the length PQ , giving your answer in simplified surd form

[3 marks]

Question 8

Differentiate with respect to x

$$y = \frac{5x^4 + 7x}{x^3}$$

[3 marks]

Question 9

(i) Expand the brackets;

$$y = \left(x^{\frac{3}{2}} - 1 \right) \left(x^{-\frac{1}{2}} + 1 \right)$$

[2 marks]

(ii) For the curve described in part (i), find $\frac{dy}{dx}$

[3 marks]

(iii) Calculate the gradient of the part (i) curve at the point where $x = 4$

[2 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk