Answer 1

$$g(x) = x^4 - 256x + 800$$

$$g'(x) = 4x^3 - 256$$

$$g'(x) = 0 \quad \text{when} \quad 4x^3 - 256 = 0$$

$$x^3 - 64 = 0$$

$$x^3 = 64$$

$$x = 4$$

$$g(4) = 4^4 - 256 \times 4 + 800$$

$$= 256 - 1024 + 800$$

$$= 32$$

$$\therefore \text{ Point is } (4, 32)$$

$$g''(x) = 12x^2$$

$$g''(4) = 192$$

As this is positive $(4, 32)$ is a local minimum

[5 marks]

Answer 2

$$y = 4x^{1.5} + \frac{9}{x^3} \qquad x \neq 0$$

$$= 4x^{1.5} + 9x^{-3}$$

$$\frac{dy}{dx} = 6x^{0.5} - 27x^{-4}$$

$$= 6\sqrt{x} - \frac{27}{x^4}$$

[3 marks]

Question 3

$$y = \sqrt{x} \qquad x > 0$$

When $x = 4$ $y = 2$ $\therefore$ tangent passes through $(4, 2)$

$$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \frac{1}{2\sqrt{x}}$$

$$When \ x = 4, \qquad \frac{dy}{dx} = \frac{1}{4}$$

Tangent: $y = mx + c$

$$2 = \frac{1}{4} \times 4 + c$$

$$c = 1$$

The equation of the tangent is $y = \frac{1}{4}x + 1$

[5 marks]

Answer 4

$$f(x) = \frac{x^3}{3} - 3x^2 + 11x$$

$$f'(x) = x^2 - 6x + 11$$

$$= (x - 3)^2 - 9 + 11$$

$$= (x - 3)^2 + 2$$

The least value of $(x - 3)^2$ is zero.
Even then, 2 is added on.
That is,

$$f'(x) > 0$$

$\therefore$ $f(x)$ is a strictly increasing function.

[4 marks]

Answer 5

(i) $(x + h)^3 = x^3 + 3x^2 h + 3x h^2 + h^3$

[2 marks]

$$f(x) = 4x^3$$

(ii) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4(x+h)^3 - 4x^3}{h}$$

$$= \lim_{h \to 0} \frac{4\left(x^3 + 3x^2 h + 3x h^2 + h^3\right) - 4x^3}{h}$$

$$= \lim_{h \to 0} \frac{4x^3 + 12x^2 h + 12x h^2 + 4h^3 - 4x^3}{h}$$

$$= \lim_{h \to 0} \frac{12x^2 h + 12x h^2 + 4h^3}{h}$$

$$= \lim_{h \to 0} \frac{h\left(12x^2 + 12xh + 4h^2\right)}{h}$$

$$= \lim_{h \to 0} 12x^2 + 12xh + 4h^2$$

$$= 12x^2 \qquad \square$$

[4 marks]

Answer 6

(i) The 84 cm of frame stretches over two sides of length y and two of length x.

In other words, $84 = 2y + 2x \Rightarrow y = 42 - x$

The area enclosed is, $A = yx$

$$= (42 - x)x$$

$$= 42x - x^2 \qquad \square$$

[3 marks]

(ii) $\frac{dA}{dx} = 42 - 2x$

For a turning point $\frac{dA}{dy} = 0 \quad \therefore \quad 42 - 2x = 0 \Rightarrow x = 21$ cm

[2 marks]

(iii) $\therefore A_{MAX} = 21 \times (42 - 21) = 441$ cm^2

[1 mark]

(iv) $\frac{d^2A}{dx^2} = -2$ That this is negative indicates a maximum $\square$

[2 marks]

Answer 7

$$y = 17x + 3x^{\frac{2}{3}} - x^2 - 80 \qquad x > 0$$

(i) $$\frac{dy}{dx} = 17 + 2x^{-\frac{1}{3}} - 2x$$

$$= 17 + \frac{2}{\sqrt[3]{x}} - 2x$$

[3 marks]

(ii) When $x = 8$,

$$y = 17 \times 8 + 3 \times 8^{\frac{2}{3}} - 8^2 - 80$$

$$= 136 + 12 - 64 - 80$$

$$= 4$$

$\therefore$ (8, 4) is on the curve C ☐

[2 marks]

(iii) When $x = 8$,

$$\frac{dy}{dx} = 17 + \frac{2}{\sqrt[3]{8}} - 2 \times 8$$

$$= 17 + 1 - 16$$

$$= 2$$

Thus $m_N = -\frac{1}{2}$

Normal: $y = mx + c$

$$4 = -\frac{1}{2} \times 8 + c$$

$$c = 8$$

The equation of the normal is thus $y = -\frac{1}{2}x + 8$

In other words, $2y + x = 16$ ☐

[4 marks]

(iv) The normal will cut the x-axis when $y = 0$, when $x = 16$
Require the distance, d, between Q(16, 0) and P(8, 4)

$$d = \sqrt{(\Delta y)^2 + (\Delta x)^2}$$

$$= \sqrt{4^2 + 8^2}$$

$$= \sqrt{80}$$

$$= 4\sqrt{5}$$

[3 marks]

Answer 8

$$y = \frac{5x^4 + 7x}{x^3}$$

$$= \frac{5x^4}{x^3} + \frac{7x}{x^3}$$

$$= 5x + 7x^{-2}$$

$$\frac{dy}{dx} = 5 - 14x^{-3}$$

$$= 5 - \frac{14}{x^3}$$

[3 marks]

Answer 9
(i)

$$y = \left(x^{\frac{3}{2}} - 1\right)\left(x^{-\frac{1}{2}} + 1\right)$$

$$= x - 1 - x^{-\frac{1}{2}} + x^{\frac{3}{2}}$$

[2 marks]

(ii)

$$\frac{dy}{dx} = 1 + \frac{1}{2}x^{-\frac{3}{2}} + \frac{3}{2}x^{\frac{1}{2}}$$

[3 marks]

(iii)

$$\frac{dy}{dx} = 1 + \frac{1}{2} \times 4^{-\frac{3}{2}} + \frac{3}{2} \times 4^{\frac{1}{2}}$$

$$= 1 + \frac{1}{2} \times \frac{1}{8} + \frac{3}{2} \times 2$$

$$= 1 + \frac{1}{16} + 3$$

$$= 4\frac{1}{16}$$

[2 marks]