2.4 Answers

2.4.1 Solution (Exercise 2.3)

Answer 1

(i) 11, 18, 25, 32, 39, 46, ...

(ii) $a = 11$, $d = 7$ and $n = 400$

$$\begin{aligned} L_n &= a + (n - 1)\,d \qquad\qquad n \geqslant 1 \\ &= 11 + (400 - 1)\,7 \\ &= 11 + 2800 - 7 \\ &= 2804 \end{aligned}$$

(iii)

$$\begin{aligned} S_n &= \frac{n}{2}\{a + L\} \qquad\qquad n \geqslant 1 \\ &= \frac{400}{2}\{11 + 2804\} \\ &= 200 \times 2815 \\ &= 563000 \end{aligned}$$

Answer 2

(a) $a = 5$, $d = 2$ and $n = 200$

$$\begin{aligned} L_n &= a + (n - 1)\,d \qquad\qquad n \geqslant 1 \\ &= 5 + (200 - 1)\,2 \\ &= 5 + 400 - 2 \\ &= 403 \text{ pence} \end{aligned}$$

(b)

$$\begin{aligned} S_n &= \frac{n}{2}\{a + L\} \qquad\qquad n \geqslant 1 \\ &= \frac{200}{2}\{5 + 403\} \\ &= 100 \times 408 \\ &= 40800 \text{ pence} \quad (\pounds 408) \end{aligned}$$

Answer 3

(a) $a = 200$, $d = 20$, $L_n = 600$, and $n = N$

$$L_n = a + (n - 1)\,d \qquad n \geqslant 1$$

$$600 = 200 + (N - 1)\,20$$

$$400 = 20N - 20$$

$$420 = 20N$$

$$N = 21$$

(b)

S_n = N° of phones to week 21 + N° of phones after week 21

$$= \frac{n}{2}\{a + L\} + 600 \times (52 - 21)$$

$$= \frac{21}{2}\{200 + 600\} + 600 \times 31$$

$$= \frac{21 \times 800}{2} + 18600$$

$$= 8400 + 18600$$

= 27000 phones

Answer 4

(a) 1960 is Year 10

$$L_{10} = 2400 \qquad L_{40} = 600$$

$$2400 = a + 9d \qquad 600 = a + 39d$$

Combine these two equations by subtraction,

$$1800 = -30d$$

$$d = -60$$

(b) Substituting this answer back into the first equation,

$$2400 = a + 9 \times (-60)$$

$$a = 2940$$

(c)

$$S_n = \frac{n}{2}\{a + L\} \qquad n \geqslant 1$$

$$= \frac{40}{2}\{2940 + 600\}$$

$$= 20 \times 3540$$

= 70800 houses

Answer 5

$$\sum_{1}^{21} (7n + 3) = 10 + 17 + 24 + \ldots + 150$$

The series described is in Arithmetic Progression with a = 10, d = 7, L = 150, n = 21

Thus,

$$\sum_{1}^{21} (7n + 3) = \frac{21}{2} \{ 10 + 150 \}$$

$$= 1680$$

Answer 6

(a)

$$25 = a + 17d \qquad\qquad 32.5 = a + 20d$$

(b) Combine these two equations by subtraction,

$$7.5 = 3d$$

$$d = 2.5$$

Substitute back into the first equation,

$$25 = a + 17 \times 2.5$$

$$a = 25 - 42.5$$

$$= -17.5 \qquad \text{as expected}$$

Doing the question this way uses the check on a to also check d

(c)

$$S_n = \frac{n}{2} \{ 2a + (n - 1) d \} \qquad n \geqslant 1$$

$$2750 = \frac{n}{2} \{-35 + 2.5n - 2.5\}$$

$$5500 = n \{-37.5 + 2.5n \}$$

$$2.5n^2 - 37.5n - 5500 = 0$$

$$n^2 - 15n - 2200 = 0$$

$$\therefore n^2 - 15n = 55 \times 40 \qquad \square$$

(d)

$$n^2 - 15n - 55 \times 40 = 0$$

$$(n - 55)(n + 40) = 0$$

$$\therefore n = 55 \qquad \text{reject } n = -40 \text{ as } n \text{ must be positive}$$

Answer 7

(a) $-3, -1, 1, \ldots$

(b) 2

(c)

$$\sum_{r=1}^{n}(2r-5) = \frac{n}{2}\{-3\times 2+(n-1)\times 2\}$$

$$= n\{-3+n-1\}$$

$$= n(n-4) \qquad \square$$