3.5 Answers

3.5.1 Solution (Example)

(a)

$$\begin{aligned} a_2 &= 5a_1 - 3 \\ 7 &= 5a_1 - 3 \\ 5a_1 &= 10 \\ a_1 &= 2 \end{aligned}$$

(b)

$$\begin{aligned} \sum_{r=1}^{4} a_r &= a_1 + a_2 + a_3 + a_4 \\ &= 2 + 7 + 32 + 157 \\ &= 198 \end{aligned}$$

3.5.2 Solution (Exercise 3.2)

Answer 1

(a)

$$u_3 = 17, \qquad u_4 = 33$$

(b)

$$\begin{aligned} u_2 &= 2u_1 - 1 \\ 9 &= 2u_1 - 1 \\ 2u_1 &= 10 \\ u_1 &= 5 \end{aligned}$$

$$\begin{aligned} \sum_{r=1}^{4} u_r &= u_1 + u_2 + u_3 + u_4 \\ &= 5 + 9 + 17 + 33 \\ &= 64 \end{aligned}$$

Answer 2

(a)

$$a_2 = \sqrt{7} \qquad a_3 = \sqrt{10}$$

(b)

$$\begin{aligned} a_4 &= \sqrt{(\sqrt{10})^2 + 3} = \sqrt{13} \\ a_5 &= \sqrt{(\sqrt{13})^2 + 3} = \sqrt{16} \\ &= 4 \end{aligned}$$

Answer 3

$$S_n = \frac{n}{2}\{ a + L \} \qquad n \geqslant 1$$

$$77 = \frac{11}{2}\{ a + 9 \}$$

$$a + 9 = 14$$

$$a = 5$$

$$L_n = a + (n - 1) d \qquad n \geqslant 1$$

$$9 = 5 + (11 - 1) d$$

$$10d = 4$$

$$d = 0.4$$

Answer 4

(a)

$$L_n = a + (n - 1) d \qquad n \geqslant 1$$

$$L_{10} = 150 + (10 - 1) \times 10$$

$$= £240$$

(b)

$$S = \frac{n}{2}\{ 2a + (n - 1)\ d \} \qquad n \geqslant 1$$

$$= \frac{20}{2}\ \{ 2 \times 150 + (20 - 1)\ \times 10$$

$$= 10\ \{ 300 + 190 \}$$

$$= £4900$$

(c)

$$S = \frac{n}{2}\{ 2a + (n - 1)\ d \} \qquad n \geqslant 1$$

$$2 \times 4900 = \frac{20}{2}\ \{ 2A + (20 - 1) \times 30 \}$$

$$9800 = 10\ \{2A + 570 \}$$

$$2A = 980 - 570$$

$$A = £205$$

Answer 5

(a)

$$a_2 = 4, \qquad a_3 = 7$$

(b)

$$a_4 = 16 \qquad and \qquad a_5 = 43$$

$$\sum_{r=1}^{5} a_r = a_1 + a_2 + a_3 + a_4 + a_5$$

$$= 3 + 4 + 7 + 16 + 43$$

$$= 73$$

Answer 6

(a)

$$L_n = a + (n - 1) d \qquad n \geqslant 1$$

$$L_8 = 150 + (8 - 1) \times 10$$

$$= 150 + 70$$

$$= 220 \text{ computers}$$

(b)

$$S = \frac{n}{2} \{ 2a + (n - 1) d \} \qquad n \geqslant 1$$

$$= \frac{14}{2} \{ 2 \times 150 + (14 - 1) \times 10$$

$$= 7 \times \{ 300 + 130 \}$$

$$= 3010$$

(c) The two arithmetic progressions are,

N° of computers : 150, 160, 170, 180, 190, ... , $\{10n + 140\}$, ...

Cost per computer : 900. 880, 860, 840, 820, ... , $\{920 - 20n\}$, ...

The stipulated condition is that,

$$920 - 20n = 3(10n + 140)$$

$$920 - 20n = 30n + 420$$

$$500 = 50n$$

$$n = 10 \text{ years}$$

$$i.e, \quad 2009$$

Answer 7

(a)

$$a_2 = 2\,a_1 - 7$$
$$a_2 = 2k - 7$$

(b)

$$\begin{aligned} a_3 &= 2\,(\,2k - 7\,) - 7 \\ &= 4k - 14 - 7 \\ &= 4k - 21 \end{aligned}$$

(c)

$$\begin{aligned} \sum_{r=1}^{4} a_r &= 43 \\ a_1 + a_2 + a_3 + a_4 &= 43 \\ k + 2k - 7 + 4k - 21 + 8k - 49 &= 43 \\ 15k - 77 &= 43 \\ 15k &= 120 \\ k &= 8 \end{aligned}$$

Answer 8

(a)

$$\begin{aligned} L_n &= a + (\,n - 1\,)\,d \qquad\qquad n \geqslant 1 \\ L_{25} &= 30 + (\,25 - 1\,)\ \times (-1.5) \\ &= -6 \end{aligned}$$

(b)

$$\begin{aligned} L_n &= a + (\,n - 1\,)\,d \qquad\qquad n \geqslant 1 \\ 0 &= 30 + (\,r - 1\,)\ \times (-1.5) \\ 0 &= 30 - 1.5r + 1.5 \\ 1.5r &= 31.5 \\ r &= 21 \end{aligned}$$

(c)

$$\begin{aligned} S &= \frac{n}{2}\{\,a + L\,\} \qquad\qquad n \geqslant 1 \\ &= \frac{21}{2}\ \{\,30 + 0\,\} \\ &= 10.5 \times 30 \\ &= 315 \end{aligned}$$