4.2 Solutions (Exercise 4.1)

Answer 1

(a)

$$x_2 = a + 5$$

(b)

$$\begin{aligned} x_3 &= a\,(a + 5) + 5 \\ &= a^2 + 5a + 5 \end{aligned}$$

(c)

$$\begin{aligned} 41 &= a^2 + 5a + 5 \\ a^2 + 5a - 36 &= 0 \\ (a + 9)\,(a - 4) &= 0 \end{aligned}$$

Either $x = -9$ or $x = 4$

Answer 2

(a)

$$\begin{aligned} S &= \frac{n}{2}\{2a + (n - 1)\,d\} \qquad n \geqslant 1 \\ &= \frac{10}{2}\{2 \times P + (10 - 1) \times 2T \\ &= 5\{2P + 18T\} \\ &= £\,(10P + 90T) \qquad \square \end{aligned}$$

(b)

$$\begin{aligned} \frac{10}{2}\{2 \times (P + 1800) + (10 - 1) \times T\} &= 10P + 90T \\ 5\{2P + 3600 + 9T\} &= 10P + 90T \\ 10P + 18000 + 45T &= 10P + 90T \\ 45T &= 18000 \\ T &= 400 \end{aligned}$$

(c)

$$\begin{aligned} L_n &= a + (n - 1)\,d \qquad n \geqslant 1 \\ 29850 &= P + 1800 + (10 - 1) \times 400 \\ 29850 &= P + 5400 \\ P &= 24450 \end{aligned}$$

Answer 3

(a)

$$a_2 = 6k$$

(b)

$$\begin{aligned} a_3 &= k\,(6k + 2) \\ &= 6k^2 + 2k \end{aligned}$$

$$\sum_{i=1}^{3} a_i = 2$$

$$\begin{aligned} a_1 + a_2 + a_3 &= 2 \\ 4 + 6k + 6k^2 + 2k &= 2 \\ 6k^2 + 8k + 2 &= 0 \\ 3k^2 + 4k + 1 &= 0 \\ (3k + 1)\,(k + 1) &= 0 \end{aligned}$$

$$\therefore\ k = -\frac{1}{3} \text{ or } k = -1$$

Answer 4

(a)

$$\begin{aligned} x_2 &= 1 \times (p + 1) \\ &= p + 1 \end{aligned}$$

(b)

$$\begin{aligned} x_3 &= (p + 1)\,(p + (p + 1)) \\ &= (p + 1)\,(2p + 1) \\ &= 2p^2 + 3p + 1 \\ &= 1 + 3p + 2p^2 \qquad \square \end{aligned}$$

(c)

$$\begin{aligned} 1 + 3p + 2p^2 &= 1 \\ 2p^2 + 3p &= 0 \\ p\,(2p + 3) &= 0 \end{aligned}$$

Either $p = 0$ (Reject as told $p \neq 0$)

$$\text{oor } p = -\frac{3}{2}$$

(d)

As $x_1 = x_2$ the sequence has period 2

$$\therefore x_{2008} = x_2 = -0.5$$

Answer 5

(a)

$$L_n = a + (n - 1)\,d \qquad n \geqslant 1$$

$$L_{20} = 140 + (20 - 1) \times 20$$

$$= 520 \text{ points}$$

(b)

$$S_n = \frac{n}{2}\{a + L\} \qquad n \geqslant 1$$

$$S_{20} = \frac{20}{2}\{140 + 520\}$$

$$= 6600 \text{ points}$$

(c)

$$S_n = \frac{n}{2}\{a + L\} \qquad n \geqslant 1$$

$$8500 = \frac{n}{2}\{300 + 700\}$$

$$17000 = n\{1000\}$$

$$n = 17 \text{ dragons}$$

Answer 6

(a)

$$a_2 = 3k + 5$$

(b)

$$a_3 = 3(3k + 5) + 5$$

$$= 9k + 15 + 5$$

$$= 9k + 20 \qquad \square$$

(c) **(i)**

$$\sum_{r=1}^{4} a_r = a_1 + a_2 + a_3 + a_4$$

$$= k + 3k + 5 + 9k + 20 + 27k + 65$$

$$= 40k + 90$$

(ii)

$$\sum_{r=1}^{4} a_r = 40k + 90$$

$$= 10(4k + 9)$$

which is divisible by 10 $\square$

Answer 7

(a)

$$S = \frac{n}{2}\{ 2a + (n - 1)\, d \} \qquad n \geqslant 1$$

$$162 = \frac{10}{2}\{2a + 9d \}$$

$$10a + 45d = 162 \qquad \square$$

(b)

$$L_n = a + (n - 1)\, d \qquad n \geqslant 1$$

$$17 = a + 5d$$

(c)

$$10a + 50d = 170$$

SUBTRACT $\quad 10a + 45d = 162$

$$5d = 8$$

$$d = 1.6$$

$$\therefore\ a = 17 - 5d$$

$$= 9$$

Answer 8

(a)

$$L_n = a + (n - 1)\, d \qquad n \geqslant 1$$

$$40.75 = a + 29d$$

(b)

$$S_n = \frac{n}{2}\{ a + L \} \qquad n \geqslant 1$$

$$1005 = \frac{30}{2}\{ a + 40.75 \}$$

$$15\ (a + 40.75) = 1005 \qquad \square$$

(c)

$$67 = a + 40.5$$

$$a = £26.25$$

$$\therefore\ d = \frac{40.75 - 26.25}{29}$$

$$d = 0.5$$

Answer 9

(a)

$$a_2 = 5k + 3$$

(b)

$$\begin{aligned} a_3 &= 5\,(\,5k + 3\,) + 3 \\ &= 25k + 15 + 3 \\ &= 25k + 18 \end{aligned}$$

(c) **(i)**

$$\begin{aligned} \sum_{r=1}^{4} a_i &= a_1 + a_2 + a_3 + a_4 \\ &= k + 5k + 3 + 25k + 18 + 125k + 93 \\ &= 156k + 114 \end{aligned}$$

(ii)

$$\begin{aligned} \sum_{r=1}^{4} a_i &= 156k + 114 \\ &= 6\,(\,26k + 19\,) \end{aligned}$$

whisch is divisible by 6 □

Answer 10

(a) The sequence is in Arithmetic Progression...

$$a = 2, \quad d = 2, \quad L = 100$$

$$\begin{aligned} L_n &= a + (\,n - 1\,)\,d \qquad n \geqslant 1 \\ 100 &= 2 + (n - 1) \times 2 \\ 98 &= 2n - 2 \\ 2n &= 100 \\ n &= 50 \end{aligned}$$

which is, perhaps, obvious

$$\begin{aligned} S_n &= \frac{n}{2}\,\{\,a + L\,\} \qquad n \geqslant 1 \\ &= \frac{50}{2}\,\{\,2 + 100\,\} \\ &= 2550 \end{aligned}$$

(b) **(i)**

$$a = k, \quad d = k, \quad L = 100$$

$$L_n = a + (n - 1)\, d \qquad n \geqslant 1$$

$$100 = k + (n - 1)\, k$$

$$100 = k + nk - k$$

$$n = \frac{100}{k}$$

(ii)

$$S_n = \frac{n}{2} \{ a + L \} \qquad n \geqslant 1$$

$$= \frac{100}{2k} \{ k + 100 \}$$

$$= 50 + \frac{5000}{k}$$

(c)

$$a = 2k + 1 \qquad d = 2k + 3$$

$$L_n = a + (n - 1)\, d \qquad n \geqslant 1$$

$$= 2k + 1 + (50 - 1)\ (2k + 3)$$

$$= 2k + 1 + 49\ (2k + 3)$$

$$= 2k + 1 + 98k + 147$$

$$= 100k + 148$$