

Lesson 5

A-Level Pure Mathematics Sequences & Series : Year 2

5.1 TEST

Algebraically, an Arithmetic Progression is a number sequence of the form;

$$a, \quad a + d, \quad a + 2d, \quad a + 3d, \quad \dots, \quad a + (n - 1) d$$

where a is the initial term

d is the common difference

n is number of terms.

The n^{th} term, L_n , is given by

$$L_n = a + (n - 1) d \quad n \geq 1$$

The sum of an AP is given by

$$S = \frac{n}{2} \{ a + L \} \quad n \geq 1$$

n times the average of the first and last terms

Or

$$S = \frac{n}{2} \{ 2a + (n - 1) d \} \quad n \geq 1$$

Question 1

A sequence of numbers, A , is described by the formula:

$$A_n = 4n - 3 \quad n \geq 1$$

(i) Write out the first six terms in the sequence.

[2 marks]

(ii) Is this sequence in Arithmetic Progression ?
Give a reason for your answer.

[2 marks]

Question 2

A series of numbers, B , is described in sigma notation as:

$$\sum_{1}^{\infty} n^2 - n$$

(i) Write out the first six terms in the series.

[2 marks]

(ii) Is this series in Arithmetic Progression ?
Give a reason for your answer.

[2 marks]

Question 3

In question 1 we had a ***sequence*** of numbers, and in question 2 a ***series*** of numbers.
What is the difference between a sequence and a series of numbers ?

[2 marks]

Question 4

How many terms are in this arithmetic series ?

$$8 + 14 + 20 + \dots + 164$$

[4 marks]

Question 5

Find the sum of this arithmetic series :

$$123 + 112 + 101 + \dots \dots + 13$$

[7 marks]

Question 6

The sum S of an arithmetic series is given by

$$S = \sum_{1}^{40} (4 + 3r)$$

(i) Write down the first six terms of the series.

[1 mark]

(ii) Find the common difference of the series.

[1 mark]

(iii) Calculate the value of S .

[4 marks]

Question 7

The fifth term of an arithmetic series is 26 and the eleventh term is 44.

(i) Find the first term and the common difference.

[7 marks]

(ii) List the first twelve terms of the arithmetic progression.

[1 mark]

Question 8

C1 Examination question from June 2008, Q7

Sue is training for a marathon. her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(a) Show that on the 4th Saturday of training she runs 11 km.

[1 mark]

(b) Find an expression, in terms of n , for the length of her training run on the n^{th} Saturday.

[2 marks]

(c) Show that the total distance she runs on Saturdays in n weeks of training is $n (n + 4)$ km.

[3 marks]

On the n^{th} Saturday Sue runs 43 km.

(d) Find the value of n .

[2 marks]

(e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.

[2 marks]

Question 9

C1 Examination question from January 2006, Q7

On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was £500 and on each following birthday the allowance was increased by £200.

- (a) Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200.

[1 mark]

- (b) Find the amount of Alice's annual allowance on her 18th birthday.

[2 marks]

- (c) Find the total of the allowances that Alice had received up to and including her 18th birthday.

[3 marks]

When the total of the allowances that Alice had received reached £32 000 the allowance stopped.

(d) Find how old Alice was when she received her last allowance.

[7 marks]

Question 10

This final question is a 'challenge' question; hard but not impossible !

An arithmetic progression contains only positive integer terms.

The sum of the first three terms is 51.

The sum of the last four terms is 332.

- (i) Show that only two arithmetic progressions satisfy these conditions.
- (ii) List the two arithmetic progressions.

[8 marks]

Examination questions are © Pearson Education Ltd

This document is Licensed for use by staff and students at **Shrewsbury School, England**

To obtain a Licence please visit www.NumberIsAll.com

© 2020 Number Is All