5.2 Solutions (Test 5.1)

Answer 1

(i)

$$1,\ 5,\ 9,\ 13,\ 17,\ 21,\ \ldots$$

(ii) Yes, $a = 1$ and $d = 4$

Answer 2

(i) The series is,

$$0 + 2 + 6 + 12 + 20 + 30 + \ldots$$

So the terms of the series are,

$$0,\ 2,\ 6,\ 12,\ 20,\ 30,\ \ldots$$

(ii) No, the differences between successive terms are not all the same.

Answer 3

A sequences is a list of numbers with successive terms separated by commas.
A series is a list of numbers with addition signs between successive terms.

Answer 4

$$\begin{aligned} L_n &= a + (n-1)\,d \qquad n \geqslant 1 \\ 164 &= 8 + (n-1) \times 6 \\ 164 &= 8 + 6n - 6 \\ 6n &= 162 \\ n &= 27 \text{ terms} \end{aligned}$$

Answer 5

$$\begin{aligned} L_n &= a + (n-1)\,d \qquad n \geqslant 1 \\ 13 &= 123 + (n-1) \times (-11) \\ 13 &= 123 - 11n + 11 \\ 11n &= 121 \\ n &= 11 \text{ terms} \end{aligned}$$

$$\begin{aligned} S &= \frac{n}{2}\{a + L\} \qquad n \geqslant 1 \\ &= \frac{11}{2}\{123 + 13\} \\ &= 748 \end{aligned}$$

Answer 6

(i)

$$S = \sum_{1}^{40} (4 + 3r)$$

$$= 7 + 10 + 13 + 16 + 19 + 22 + ...$$

So the first six terms are,

$$7,\ 10,\ 13,\ 16,\ 19,\ 22$$

(ii) $d = 3$

(iii) $a = 7 \quad n = 40$

$$S = \frac{n}{2}\{ 2a + (n - 1)\, d \} \qquad n \geqslant 1$$

$$= \frac{40}{2}\{ 2 \times 7 + 39 \times 3 \}$$

$$= 2620$$

Answer 7

(i)

$$a + 10d = 44$$

$$\text{SUBTRACT} \qquad a + 4d = 26$$

$$\text{———————————————————}$$

$$6d = 18$$

$$d = 3 \qquad \therefore\ a = 14$$

(ii)

$$14,\ 17,\ 20,\ 23,\ 26,\ 29,\ 32,\ 35,\ 38,\ 41,\ 44,\ 47,\ ...$$

Answer 8

(a)

$$L_n = a + (n - 1)\, d \qquad n \geqslant 1$$

$$= 5 + (4 - 1) \times 2$$

$$= 11 \text{ km} \qquad \square$$

(b)

$$L_n = 5 + (n - 1)\, 2$$

$$= 3 + 2n$$

(c)

$$S = \frac{n}{2}\{ 2a + (n - 1)\, d \} \qquad n \geqslant 1$$

$$= \frac{n}{2}\{ 2 \times 5 + (n - 1) \times 2 \}$$

$$= n\,(n + 4) \qquad \square$$

(d)

$$3 + 2n = 43$$

$$n = 20$$

(e)

$$n\,(n + 4) = 20 \times 24$$

$$= 480 \text{ km}$$

Answer 9

(a)

$$S = \frac{n}{2}\{2a + (n - 1)\,d\} \qquad n \geqslant 1$$

$$= \frac{2}{2}\{2 \times 500 + (2 - 1) \times 200\}$$

$$= £1200 \qquad \square$$

(b)

$$L_n = a + (n - 1)\,d \qquad n \geqslant 1$$

$$L_{18} = 500 + (8 - 1) \times 200$$

$$= £1900$$

(c)

$$S = \frac{n}{2}\{2a + (n - 1)\,d\} \qquad n \geqslant 1$$

$$= \frac{8}{2}\{2 \times 500 + (8 - 1) \times 200\}$$

$$= £9600$$

(d)

$$S = \frac{n}{2}\{2a + (n - 1)\,d\} \qquad n \geqslant 1$$

$$32000 = \frac{n}{2}\{2 \times 500 + (n - 1) \times 200\}$$

$$64000 = n\{1000 + 200n - 200\}$$

$$64000 = 800n + 200n^2$$

$$n^2 + 4n - 320 = 0 \qquad \text{divide through by } 200$$

$$(n + 2)^2 - 324 = 0 \qquad \text{completing the square}$$

$$(n + 2)^2 = 324$$

$$n + 2 = \pm 18$$

$$n = 16 \qquad (\text{Reject } n = -20)$$

$\therefore$ Alice is aged 26

Answer 10

The sum of the first three terms,

$$a + a + d + a + 2d = 51$$
$$3a + 3d = 51$$
$$a + d = 17$$
$$\therefore d \leqslant 16 \qquad \text{as sequence only has positive integer terms}$$

The sum of the last four terms,

$$a + (n-4)d + a + (n-3)d + a + (n-2)d + a + (n-1)d = 332$$
$$4a + 4nd - 10d = 332$$
$$2a + 2nd - 5d = 166$$

Substitute $a = 17 - d$ to get,

$$2(17 - d) + 2nd - 5d = 166$$
$$2nd - 7d = 132$$
$$d(2n - 7) = 2 \times 2 \times 3 \times 11$$

Work through the possibilities,

d	2	4	6	11	12
$2n$ - 7	66	33	22	12	11
n	36.5	20	14.5	9.5	9

Notice that for n to be an integer, $2n$ - 7 has to be odd

As n must be an integer, two possibilities need to be listed,

13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89

5, 17, 29, 41, 53, 65, 77, 89, 101