6.5 Solutions (to 6.4 Exercise)

Answer 1

All terms in $AP\{6,10\}$ are of the form.

$$y = 6 + 10n, \qquad for \quad n = 0, 1, 2, 3, \ldots$$

$$y = 2\,(3 + 5n)$$

This shows that these numbers are always divisible by 2.

The first term, 6, is not prime, and so no term in $AP\{6,10\}$ is prime ☐

Answer 2

$AP\{7,10\}$ = 7, 17, 27, 37, 47, 57, 67, 77, 87, 97, ...

So any of the numbers

- 27 (divides by 3)
- 57 (divides by 3 and 19)
- 77 (divides by 7 and 11)
- 87 (divides by 3 and 29)

... prove George is wrong.

Other bigger numbers could also be used as answers.

Answer 3

$AP\{5,10\}$ = 5, 15, 25, 35, 45, 55, 65, 75. 85, 95, ...

The only prime is 5

Answer 4

$AP\{2,10\}$ = 2, 12, 22, 32, 42, 52, 62, 72, 82, 92, ...

The first number, 2, is prime

Answer 5

$AP\{1,6\}$ and $AP\{5,6\}$

Answer 6

All terms in $AP\{4,6\}$ are of the form.

$$y = 4 + 6n, \qquad for \quad n = 0, 1, 2, 3, \ldots$$

$$y = 2\,(2 + 3n)$$

This shows that these numbers are always divisible by 2.

The first term, 4, is not prime, and so no term in $AP\{4,6\}$ is prime ☐

Answer 7

Either 1 or 5

Answer 8

x is in $AP\{2,6\}$ which has terms which all divide by 2

The only prime is 2 so either $x = 2$ or it is composite.

Answer 9

$y = 0 + 7n$		7													
$y = 1 + 7n$					29		43				71				
$y = 2 + 7n$	2			23		37						79			
$y = 3 + 7n$	3		17		31				59		73				101
$y = 4 + 7n$		11						53		67					
$y = 5 + 7n$	5		19				47		61				89		
$y = 6 + 7n$		13				41						83		97	

It's not a good prime trapper as, other than 7, the primes can be in any six of the seven rows.

Answer 10

1, 3, 5 and 7 are coprime to 8

Answer 11

$AP\{1,8\}$, $AP\{3,8\}$, $AP\{5,8\}$ and $AP\{7,8\}$ will each contain infinitely many primes.

Answer 12

$AP\{1,9\}$, $AP\{2,9\}$, $AP\{4,9\}$, $AP\{5,9\}$, $AP\{7,9\}$ and $AP\{8,9\}$ will each contain infinitely many primes.

Answer 13

(i) $a = 1, 3, 5, 7, 9, 11, 13, 15$

$AP\{3,16\} = 3, 19, 35, 51, 67, 83, 99, 115, 131, \ldots$

(ii) 35, 51, 99, 115, ...

(iii) 3, 19, 67, 83, 131, ...

Answer 14

(i)

$$\begin{aligned} \phi\left(11^1\right) &= 11^0(11 - 1) \\ &= 10 \end{aligned}$$

(ii)

$$\begin{aligned} \phi(6) &= \phi(2 \times 3) \\ &= \phi\left(2^1\right) \times \phi\left(3^1\right) \\ &= 2^0 \times 1 \times 3^0 \times 2 \\ &= 2 \end{aligned}$$

(iii)

$$\phi(8) = \phi\left(2^3\right)$$
$$= 4$$

(iv)

$$\begin{aligned}\phi(30) &= \phi\left(2 \times 3 \times 5\right)\\ &= \phi(2) \times \phi(3) \times \phi(5)\\ &= 2^0 \times 1 \times 3^0 \times 2 \times 5^0 \times 4\\ &= 8\end{aligned}$$

Answer 15

Some values of the totient function are shown in the table below,

+	0	1	2	3	4	5	6	7	8	9
0	-	1	1	2	2	4	2	6	4	6
10	4	10	4	12	6	8	8	16	6	18
20	8	12	10	22	8	20	12	18	12	28
30	8	30	16	20	16	24	12	36	18	24
40	16	40	12	42	20	24	22	46	16	42
50	20	32	24	52	18	40	24	36	28	58
60	16	60	30	36	32	48	20	66	32	44
70	24	70	24	72	36	40	36	60	24	78
80	32	54	40	82	24	64	42	56	40	88
90	24	72	44	60	46	72	32	96	42	60
100	40	100	32	102	48	48	52	106	36	108
110	40	72	48	112	36	88	56	72	58	96
120	32	110	60	80	60	100	36	126	64	84
130	48	130	40	108	66	72	64	136	44	138
140	48	92	70	120	48	112	72	84	72	148
150	40	150	72	96	60	120	48	156	78	104
160	64	132	54	162	80	80	82	166	48	156
170	64	108	84	172	56	120	80	116	88	178
180	48	180	72	120	88	144	60	160	92	108
190	72	190	64	192	96	96	84	196	60	198

Good traps are number grids of width 6 on 33% and 30 on 27%

A quick search found 420 on 23% and 4620 on 21%