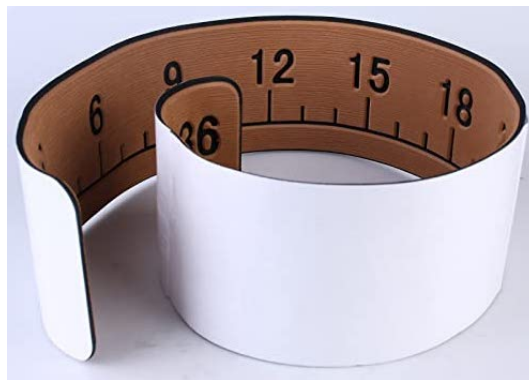
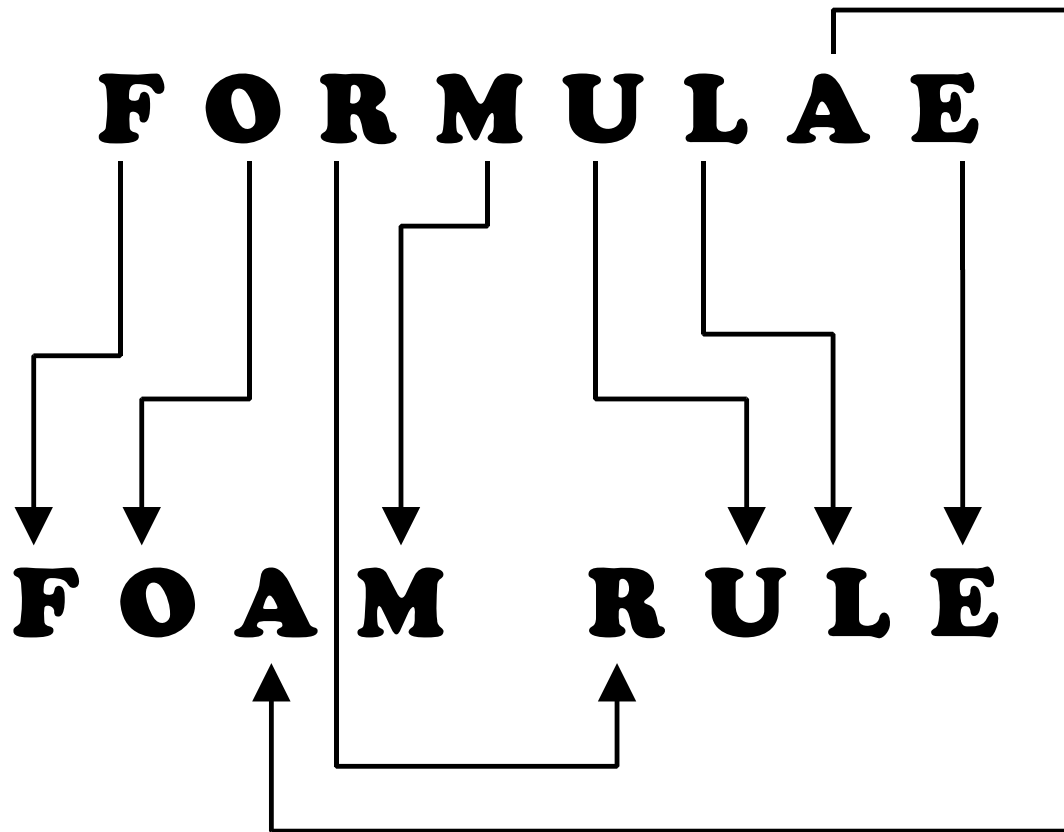


# REARRANGING



*In algebra, the golden rule  
for manipulating equations*

**“DO THE SAME TO THE WHOLE OF BOTH SIDES”**

## Lesson 1

### GCSE Mathematics Algebra, Rearranging Formulae

#### 1.1 Introduction

From London, William is talking over a phone to his friend Dwain in New York. Dwain tells William that the temperature in New York is  $59^{\circ}\text{F}$  (Fahrenheit). William is only familiar with temperatures being expressed in  $^{\circ}\text{C}$  (Centigrade). Fortunately, he knows a formula to convert from Fahrenheit to Centigrade.

$$C = \frac{5(F - 32)}{9}$$

In this formula we say that  $C$  is the subject.

William proceeds by substitution in the number 59 in place of  $F$ .

$$\begin{aligned} &= \frac{5(59 - 32)}{9} \\ &= \frac{5 \times 27}{9} \\ &= 15^{\circ}\text{C} \end{aligned}$$

In continuing the conversation, William tells Dwain that it is much colder in London, "The temperature in London is a cold  $-5^{\circ}\text{C}$ ". Dwain immediately wants to know what this is in  $^{\circ}\text{F}$ .

William does not know the formula with  $F$  as the subject.

But he can work it out.

He can **change the subject** of the formula.

|                                 |                           |                                 |
|---------------------------------|---------------------------|---------------------------------|
|                                 | $C = \frac{5(F - 32)}{9}$ |                                 |
| <i>multiply both sides by 9</i> | $9C = 5(F - 32)$          | <i>multiply both sides by 9</i> |
| <i>divide both sides by 5</i>   | $\frac{9C}{5} = F - 32$   | <i>divide both sides by 5</i>   |
| <i>add 32 to both sides</i>     | $\frac{9C}{5} + 32 = F$   | <i>add 32 to both sides</i>     |
|                                 | $F = \frac{9C}{5} + 32$   |                                 |

William now inserts London's  $5^{\circ}\text{C}$  into this rearranged formula.

$$\begin{aligned} F &= \frac{9C}{5} + 32 \\ &= \frac{9 \times (-5)}{5} + 32 \\ &= -9 + 32 \\ &= 21^{\circ}\text{F} \end{aligned}$$

|          |                      |                      |
|----------|----------------------|----------------------|
| New York | $-5^{\circ}\text{C}$ | $15^{\circ}\text{C}$ |
| London   | $21^{\circ}\text{F}$ | $59^{\circ}\text{F}$ |

## 1.2 The Golden Rule

*In algebra, there is one golden rule for manipulating all equations*  
**“DO THE SAME TO THE WHOLE OF BOTH SIDES”**

## 1.3 Example

**Question:**

*Showing full working*, make  $x$  the subject of the equation,

$$3dx - 5m = 4$$

**Solution:**

|                                |                         |                                |
|--------------------------------|-------------------------|--------------------------------|
| <i>add 5m to both sides</i>    | $3dx - 5m = 4$          | <i>add 5m to both sides</i>    |
| <i>divide both sides by 3d</i> | $3dx = 4 + 5m$          | <i>divide both sides by 3d</i> |
|                                | $x = \frac{4 + 5m}{3d}$ |                                |

**More succinctly:**

|        |                         |        |
|--------|-------------------------|--------|
| $+ 5m$ | $3dx - 5m = 4$          | $+ 5m$ |
| $/ 3d$ | $3dx = 4 + 5m$          | $/ 3d$ |
|        | $x = \frac{4 + 5m}{3d}$ |        |

Note the use of “/” for “divide”

This is because when projected onto a white board  $\div$  is easily mistaken for  $+$

## 1.4 Example

**Question:**

*Showing full working*, make  $x$  the subject of the equation,

$$7xw + 8 = 3k$$

**Solution:**

### 1.5 Exercise

Take care over how you set out your solutions, as demonstrated by the examples.

#### Question 1

*Showing full working*, make  $x$  the subject of the equation,

$$ax - b = 7$$

#### Question 2

*Showing full working*, make  $x$  the subject of the equation,

$$3c + wx = pqr$$

**Question 3**

*Showing full working*, make  $x$  the subject of the equation,

$$4x - vw = 9$$

**Question 4**

*Showing full working*, make  $x$  the subject of the equation,

$$5 + 4bx + z = a$$

**Question 5**

**Showing full working**, by first adding  $y^2$  to both sides, make  $x$  the subject of,

$$7x - y^2 = 4z + 1$$

**Question 6**

**Showing full working**, by first expanding the brackets, make  $x$  the subject of,

$$4(2x + y) = 7$$

**Question 7**

**Showing full working**, by first expanding the brackets, make  $x$  the subject of,

$$a(3x + b) = c$$

**Question 8**

**Showing full working**, by first subtracting  $y$  from both sides, make  $x$  the subject of,

$$\frac{x}{7} + y = 3z$$

Write your answer without any brackets.

**Question 9**

**Showing full working**, by first subtracting 11 from both sides, make  $x$  the subject of,

$$11 + \frac{4x}{5} = z$$

Write your answer without any brackets.

**Question 10**

**Showing full working**, by first adding 7 to both sides, make  $x$  the subject of,

$$\frac{x + p^2}{q} - 7 = R$$

## 1.6 Quick 'In your head' Questions

### GCSE Mathematics Algebra, Rearranging Formulae

If there is just one thing to be done to both sides to get from a question to the answer, it is acceptable to just write down the answer with no working.

#### Question 1

Make  $x$  the subject

$$x + 8 = k$$

#### Question 2

Make  $x$  the subject

$$5x = m$$

#### Question 3

Make  $x$  the subject

$$wx = 9$$

#### Question 4

Make  $x$  the subject

$$\frac{x}{6} = f$$

#### Question 5

Make  $x$  the subject

$$x - 6ws = p^2$$

**Question 6**

Make  $x$  the subject

$$x - \frac{3}{2} = b$$

**Question 7**

Make  $x$  the subject

$$x - 7a = c$$

**Question 8**

Make  $x$  the subject

$$x ( 4 + p ) = z$$

**Question 9**

Make  $x$  the subject

$$\frac{x}{(v + 2)} = h$$

**Question 10**

Make  $x$  the subject

$$9 + x = h^2 + u - t$$

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## 1.7 Homework

### GCSE Mathematics Algebra, Rearranging Formulae

#### Question 1

*Showing full working*, make  $x$  the subject

$$Hx - 5b = f$$

#### Question 2

*Showing full working*, make  $x$  the subject

$$7k + 3x = 6e - 1$$

**Question 3**

*Showing full working*, by first expanding the brackets, make  $x$  the subject

$$7(3x + z) = 4g$$

**Question 4**

*Showing full working*, make  $x$  the subject

$$\frac{5x}{2} + 4 = 3k$$

Write your answer without any brackets.

**Question 5**

Make  $x$  the subject

$$x - 9 = 2k$$

There is no need to show any working.

**Question 6**

Make  $x$  the subject

$$5x = 3m^2$$

There is no need to show any working.

**Question 7**

Make  $x$  the subject

$$\frac{x}{13} = uvw$$

There is no need to show any working.

**Question 8**

Make  $x$  the subject

$$x(2h + c) = s$$

There is no need to show any working.

## 1.8 Answers

### GCSE Mathematics Algebra, Rearranging Formulae

#### 1.8.1 Solutions to Example ( 1.4 Example )

$$\begin{array}{ccc} -8 & \left| \begin{array}{l} 7xw + 8 = 3k \\ 7xw = 3k - 8 \\ x = \frac{3k - 8}{7w} \end{array} \right| & -8 \\ \div 7w & & \div 7w \end{array}$$

#### 1.8.2 Solutions to Exercise ( 1.5 Exercise )

##### Answer 1

$$x = \frac{7 + b}{a}$$

##### Answer 2

$$x = \frac{pqr - 3c}{w}$$

##### Answer 3

$$x = \frac{9 + vw}{4}$$

##### Answer 4

$$x = \frac{a - z - 5}{4b}$$

##### Answer 5

$$x = \frac{4z + 1 + y^2}{7}$$

##### Answer 6

$$x = \frac{7 - 4y}{8}$$

##### Answer 7

$$x = \frac{c - ab}{3a}$$

##### Answer 8

$$x = 21z - 7y$$

##### Answer 9

$$x = \frac{5z - 55}{4}$$

##### Answer 10

$$x = q(R + 7) - p^2$$

$$\text{or } x = qR + 7q - p^2$$

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### 1.8.3 Solutions to In your Head (1.6 Quick 'In your head' Questions)

#### GCSE Mathematics Algebra, Rearranging Formulae

##### Answer 1

$$x = k - 8$$

##### Answer 2

$$x = \frac{m}{5}$$

##### Answer 3

$$x = \frac{9}{w}$$

##### Answer 4

$$x = 6f$$

##### Answer 5

$$x = p^2 + 6ws$$

##### Answer 6

$$x = b + \frac{3}{2}$$

##### Answer 7

$$x = c + 7a$$

##### Answer 8

$$x = \frac{z}{4 + p}$$

##### Answer 9

$$x = h(v + 2)$$

$$\text{or } x = hv + 2h$$

##### Answer 10

$$x = h^2 + u - t - 9$$

### 1.8.4 Solutions to Homework ( 1.7 Homework )

### GCSE Mathematics Algebra, Rearranging Formulae

#### Answer 1

$$x = \frac{f + 5b}{H}$$

#### Answer 2

$$x = \frac{6e - 1 - 7k}{3}$$

#### Answer 3

$$x = \frac{4g - 7z}{21}$$

#### Answer 4

$$x = \frac{6k - 8}{5}$$

#### Answer 5

$$x = 2k + 9$$

#### Answer 6

$$x = \frac{3m^2}{5}$$

#### Answer 7

$$x = 13uvw$$

#### Answer 8

$$x = \frac{s}{2h + c}$$

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## Lesson 2

### GCSE Mathematics Algebra, Rearranging Formulae

#### 2.1 Reciprocal

The reciprocal of  $\frac{4}{7}$  is  $\frac{7}{4}$

The reciprocal of  $\frac{6}{x}$  is  $\frac{x}{6}$

The reciprocal of  $\frac{1}{2}$  is  $\frac{2}{1}$  (which you could write as just 2)

#### 2.2 Quick Questions

Remembering to

**“DO THE SAME TO THE WHOLE OF BOTH SIDES”**

write down the reciprocal of,

( i )  $\frac{8}{3}$

( ii )  $\frac{7}{x}$

( iii )  $\frac{1}{7}$

( iv )  $\frac{13}{x}$

( v )  $\frac{x}{5}$

( vi )  $\frac{3x}{2}$

( vii )  $\frac{8e}{11}$

( viii )  $\frac{w}{3x}$

( ix )  $\frac{m}{4pw}$

( x ) 4

( xi )  $x$

( xii )  $8m$

( xiii )  $\frac{4}{p + q}$

( xiv )  $\frac{1}{m^2 + n}$

( xv )  $a + b$

## 2.3 Taking the reciprocal of both sides

### Example

*Showing full working*, make  $x$  the subject;

$$\frac{3}{5x} + 2m = p$$

## 2.4 Exercise

### Question 1

*Showing full working*, make  $x$  the subject;

$$\frac{1}{x} - c = 5ab$$

### Question 2

*Showing full working*, make  $x$  the subject;

$$\frac{7}{x} + 2pq = w$$

**Question 3**

*Showing full working*, make  $x$  the subject;

$$\frac{5k}{x} + z = 6$$

**Question 4**

*Showing full working*, by first taking the reciprocal of both sides, make  $x$  the subject;

$$\frac{1}{x + 3p} = \frac{4zk}{7m}$$

**Question 5**

**Showing full working**, by first adding 11 to both sides, make  $x$  the subject;

$$\frac{2 + w}{x} - 11 = k$$

**Question 6**

**Showing full working**, by first taking the reciprocal of both sides, make  $x$  the subject;

$$\frac{1}{x(2w + v)} = m^2$$

**Question 7**

**Showing full working**, by first subtracting  $5u$  from both sides, make  $x$  the subject;

$$\frac{8}{7x} + 5u = 2g$$

## 2.5 Quick 'In your head' Questions

### GCSE Mathematics Algebra, Rearranging Formulae

If there is just one thing to be done to both sides to get from a question to the answer, it is acceptable to just write down the answer with no working.

#### Question 1

Make  $x$  the subject

$$x - 11 = 2m$$

#### Question 2

Make  $x$  the subject

$$7x = w$$

#### Question 3

Make  $x$  the subject

$$kx = z$$

#### Question 4

Make  $x$  the subject

$$\frac{x}{5} = s$$

#### Question 5

Make  $x$  the subject

$$x - 6s^2 = 3$$

**Question 6**

Make  $x$  the subject

$$\frac{1}{x} = \frac{5}{3}$$

**Question 7**

Make  $x$  the subject

$$\frac{1}{x} = c^2 + 4$$

**Question 8**

Make  $x$  the subject

$$\frac{1}{x} = \frac{5v + 2}{p + q}$$

**Question 9**

Make  $x$  the subject

$$\frac{x}{(w^2 + 2k)} = h$$

**Question 10**

Make  $x$  the subject

$$\sqrt{7z} + x = h^2$$

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## 2.6 Answers

### GCSE Mathematics Algebra, Rearranging Formulae

#### 2.6.1 Solutions to Quick Questions 2.2

|          |                 |          |                |         |                 |
|----------|-----------------|----------|----------------|---------|-----------------|
| ( i )    | $\frac{3}{8}$   | ( ii )   | $\frac{x}{7}$  | ( iii ) | 7               |
| ( iv )   | $\frac{x}{13}$  | ( v )    | $\frac{5}{x}$  | ( vi )  | $\frac{2}{3x}$  |
| ( vii )  | $\frac{11}{8e}$ | ( viii ) | $\frac{3x}{w}$ | ( ix )  | $\frac{4pw}{m}$ |
| ( x )    | $\frac{1}{4}$   | ( xi )   | $\frac{1}{x}$  | ( xii ) | $\frac{1}{8m}$  |
| ( xiii ) | $\frac{p+q}{4}$ | ( xiv )  | $m^2 + n$      | ( xv )  | $\frac{1}{a+b}$ |

#### 2.6.2 Solution to Example 2.3

$$\begin{array}{ccc}
 & \left| \frac{3}{5x} + 2m = p \right| & \\
 -2m & & -2m \\
 & \left| \frac{3}{5x} = p - 2m \right| & \\
 \updownarrow & & \updownarrow \\
 & \left| \frac{5x}{3} = \frac{1}{p-2m} \right| & \\
 \times 3 & & \times 3 \\
 & \left| 5x = \frac{3}{p-2m} \right| & \\
 / 5 & & / 5 \\
 & \left| x = \frac{3}{5(p-2m)} \right| &
 \end{array}$$

### 2.6.3 Solution to Exercise 2.4

#### Answer 1

$$x = \frac{1}{5ab + c}$$

#### Answer 2

$$x = \frac{7}{w - 2pq}$$

#### Answer 3

$$x = \frac{5k}{6 - z}$$

#### Answer 4

$$x = \frac{7m}{4zk} - 3p$$

#### Answer 5

$$x = \frac{2 + w}{k + 11}$$

#### Answer 6

$$x = \frac{1}{m^2 (2w + v)}$$

#### Answer 7

$$x = \frac{8}{7(2g - 5u)}$$

## 2.6.4 Solution to 'In your head' 2.5

### GCSE Mathematics Algebra, Rearranging Formulae

#### Answer 1

$$x = 2m + 11$$

#### Answer 2

$$x = \frac{w}{7}$$

#### Answer 3

$$x = \frac{z}{k}$$

#### Answer 4

$$x = 5s$$

#### Answer 5

$$x = 3 + 6s^2$$

#### Answer 6

$$x = \frac{3}{5}$$

#### Answer 7

$$x = \frac{1}{c^2 + 4}$$

#### Answer 8

$$x = \frac{p + q}{5v + 2}$$

#### Answer 9

$$x = h(w^2 + 2k)$$

#### Answer 10

$$x = h^2 - \sqrt{7z}$$

$$\text{or } x = hw^2 + 2hk$$

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## Lesson 3

### GCSE Mathematics Algebra, Rearranging Formulae

#### 3.1 Drill



Photograph by Martin Hansen, January 2025

#### Question 1

In algebra, there is one golden rule for manipulating equations.  
What is that rule ?

#### Question 2

Quick 'In your head' Questions.

In each of the following, there is one manipulation that takes you from the question to the answer and so there is no need for any working.

( i )     Make  $x$  the subject

$$x - 4.5 = m$$

( ii )     Make  $x$  the subject

$$15x = h$$

( iii )     Make  $x$  the subject

$$\frac{x}{8} = c$$

( iv )    Make x the subject

$$\frac{1}{x} = \frac{11}{8}$$

**Question 3**

Write down the reciprocal of each of the following;

( i )     $\frac{8}{x}$

( ii )     $\frac{2w}{3x^2}$

( iii )     $\frac{m + 3n}{4k}$

( iv )    3

( v )    x

( vi )     $5k + p$

( vii )     $\frac{7}{p + q}$

( viii )     $\frac{1}{7}$

( ix )     $\frac{1}{x}$

**Question 4**

*Showing full working*, make x the subject

$$mx - 8n = 3k$$

**Question 5**

*Showing full working*, make  $x$  the subject

$$7x - 8 = z^2 + y$$

**Question 6**

*Showing full working*, by first expanding the brackets, make  $x$  the subject

$$5 ( 3x + y ) = 11$$

**Question 7**

**Showing full working**, by first subtracting  $11a$  from both sides, make  $x$  the subject

$$\frac{x}{5} + 11a = 4c$$

Write your answer without any brackets.

**Question 8**

**Showing full working**, make  $m$  the subject

$$5m + v = \sqrt{c}$$

**Question 9**

Quick 'In your head' Questions.

In each of the following, there is one manipulation that takes you from the question to the answer and so there is no need for any working.

( i )      Make  $x$  the subject

$$4mx = v^2 + c$$

( ii )      Make  $x$  the subject

$$\frac{x}{8\sqrt{p}} = f$$

( iii )      Make  $x$  the subject

$$\frac{1}{x} = \frac{k^2 + 2}{m + \sqrt{n}}$$

( iv )      Make  $x$  the subject

$$\frac{1}{x} = a + b - 5$$

**Question 10**

Without any working, make  $w$  the subject of the formula

$$w - 4pq = m^2$$

**Question 11**

**Showing full working**, by first subtracting  $3g$  from both sides, make  $x$  the subject;

$$\frac{5}{x} + 3g = 7k$$

**Question 12**

**Showing full working**, make  $x$  the subject;

$$\frac{7f}{x} - 13 = h$$

**Question 13**

**Showing full working**, by first taking the reciprocal of both sides, make  $x$  the subject;

$$\frac{1}{x + 5pq} = \frac{a}{7b}$$

**Question 14**

**Showing full working**, by first adding 42 to both sides, make  $x$  the subject;

$$\frac{1 + m}{x} - 42 = u$$

**Question 15**

*Showing full working*, by first subtracting  $3h$  from both sides, make  $x$  the subject;

$$\frac{4}{3z^2 + x} + 3h = g - 1$$

### 3.2 Solutions (3.1 Practice)

#### GCSE Mathematics Algebra, Rearranging Formulae

##### Answer 1

Do the same to the whole of both sides of the equation

##### Answer 2

( i )

$$x = m + 4.5$$

( iii )

$$x = 8c$$

( ii )

$$x = \frac{h}{15}$$

( iv )

$$x = \frac{8}{11}$$

##### Answer 3

( i )  $\frac{x}{8}$

( ii )  $\frac{3x^2}{2w}$

( iii )  $\frac{4k}{m + 3n}$

( iv )  $\frac{1}{3}$

( v )  $\frac{1}{x}$

( vi )  $\frac{1}{5k + p}$

( vii )  $\frac{p + q}{7}$

( viii ) 7

( ix ) x

##### Answer 4

$$x = \frac{3k + 8n}{m}$$

##### Answer 5

$$x = \frac{z^2 + y + 8}{7}$$

##### Answer 6

$$x = \frac{11 - 5y}{15}$$

##### Answer 7

$$x = 20c - 55a$$

##### Answer 8

$$m = \frac{\sqrt{c} - v}{5}$$

##### Answer 9

( i )

$$x = \frac{v^2 + c}{4m}$$

( iii )

$$x = \frac{m + \sqrt{n}}{k^2 + 2}$$

( ii )

$$x = 8\sqrt{p} f$$

( iv )

$$x = \frac{1}{a + b - 5}$$

**Answer 10**

$$w = m^2 + 4pq$$

**Answer 11**

$$x = \frac{5}{7k - 3g}$$

**Answer 12**

$$x = \frac{7f}{h + 13}$$

**Answer 13**

$$x = \frac{7b}{a} - 5pq$$

**Answer 14**

$$x = \frac{1 + m}{u + 42}$$

**Answer 15**

$$x = \frac{4}{g - 1 - 3h} - 3z^2$$

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## 4.1 Optional Exit (Test or Consolidation)



Marks Available : 58

**Question 1**

Quick 'In your head' Questions.

In each of the following, there is one manipulation that takes you from the question to the answer and so there is no need for any working.

( i )      Make  $x$  the subject;       $x + 17 = \sqrt{m}$

[ 1 mark ]

( ii )      Make  $x$  the subject;       $23x = k$

[ 1 mark ]

( iii )      Make  $x$  the subject;       $\frac{x}{5} = v^2$

[ 1 mark ]

( iv )      Make  $x$  the subject;       $\frac{1}{x} = \frac{11}{8}$

[ 1 mark ]

**Question 2**

Write down the reciprocal of each of the following;

( i )      $\frac{5}{3}$

( ii )      $\frac{4x^2}{\sin(x)}$

( iii )     7

( iv )      $\frac{1}{2}$

( v )      $\frac{1}{\sqrt{5}}$

( vi )      $\pi$

( vii )      $8 + 3k$

( viii )      $\frac{3 + v^2}{5p}$

( ix )      $\frac{2 + \arcsin(x)}{\pi}$

[ 9 marks ]

**Question 4**

*Showing full working*, make  $x$  the subject

$$ax - 8\sqrt{n^2 + 1} = 5k$$

[ 3 marks ]

**Question 5**

*Showing full working*, make  $x$  the subject

$$\sqrt{3}x - 7 = \sin(z) + y$$

[ 3 marks ]

**Question 6**

*Showing full working*, by first expanding the brackets, make  $x$  the subject

$$6(4x + p) = 13$$

[ 3 marks ]

**Question 7**

**Showing full working**, by first subtracting  $12a$  from both sides, make  $x$  the subject

$$\frac{x}{4} + 12a = 7c$$

Write your answer without any brackets.

[ 3 marks ]

**Question 8**

**Showing full working**, make  $p$  the subject

$$5p + 4v = \sqrt{\sin(m)}$$

[ 3 marks ]

**Question 9**

Quick 'In your head' Questions.

In each of the following, there is one manipulation that takes you from the question to the answer and so there is no need for any working.

( i )      Make  $x$  the subject

$$x + m^2 = v^2 + \frac{1}{c}$$

[ 1 mark ]

( ii )      Make  $x$  the subject

$$\frac{x}{8 \cos(\sqrt{k})} = f$$

[ 1 mark ]

( iii )      Make  $x$  the subject

$$\frac{1}{x} = \frac{m + 3\sqrt{k}}{z + 8}$$

[ 1 mark ]

( iv )      Make  $x$  the subject

$$\frac{1}{x} = a^2 + b^2 - 5$$

[ 1 mark ]

**Question 10**

Without any working, make  $w$  the subject of the formula

$$w - \frac{2v}{p} = m^2$$

[ 2 marks ]

**Question 11**

*Showing full working*, by first subtracting  $5k$  from both sides, make  $x$  the subject;

$$\frac{4}{x} + 5k = 7m$$

[ 3 marks ]

**Question 12**

*Showing full working*, make  $x$  the subject;

$$\frac{7f}{x} + 4 = \sin(w)$$

[ 3 marks ]

**Question 13**

*Showing full working*, by first taking the reciprocal of both sides, make  $x$  the subject;

$$\frac{1}{x + 5p} = \frac{a^2}{7 + b}$$

[ 3 marks ]

**Question 14**

*Showing full working*, by first adding  $4km$  to both sides, make  $x$  the subject;

$$\frac{3 + m}{x} - 4km = u^2$$

[ 3 marks ]

**Question 15**

**Showing full working**, by first adding  $3h$  to both sides, make  $x$  the subject;

$$\frac{4}{5z^3 + x} - 3h = g - 1$$

[ 3 marks ]

**Question 16 (Challenge !)**

**Showing full working**, by first squaring both sides, make  $x$  the subject;

$$\sqrt{x + 1} = z + 3$$

You may leave brackets in your answer.

[ 3 marks ]

**Question 17** (Challenge !)

**Showing full working**, by first square rooting both sides, make  $x$  the subject;

$$(x + 5)^2 = m$$

[ 3 marks ]

**Question 18** (Challenge !)

**Showing full working**, by first taking the reciprocal of both sides, make  $x$  the subject;

$$\frac{1}{\sqrt{x}} = \frac{3}{8}$$

[ 3 marks ]

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## 4.2 Solutions (4.1 Optional Exit)

### GCSE Mathematics Algebra, Rearranging Formulae

#### Answer 1

$$(i) \quad x = \sqrt{m} - 17$$

$$(ii) \quad x = \frac{k}{23}$$

$$(iii) \quad x = 5v^2$$

$$(iv) \quad x = \frac{8}{11}$$

[ 4 marks ]

#### Answer 2

$$(i) \quad \frac{3}{5}$$

$$(ii) \quad \frac{\sin(x)}{4x^2}$$

$$(iii) \quad \frac{1}{7}$$

$$(iv) \quad 2$$

$$(v) \quad \sqrt{5}$$

$$(vi) \quad \frac{1}{\pi}$$

$$(vii) \quad \frac{1}{8 + 3k}$$

$$(viii) \quad \frac{5p}{3 + v^2}$$

$$(ix) \quad \frac{\pi}{2 + \arcsin(x)}$$

[ 9 marks ]

#### Answer 4

$$ax - 8\sqrt{n^2 + 1} = 5k$$

$$ax = 5k + 8\sqrt{n^2 + 1}$$

$$x = \frac{5k + 8\sqrt{n^2 + 1}}{a}$$

#### Answer 5

$$\sqrt{3}x - 7 = \sin(z) + y$$

$$\sqrt{3}x = \sin(z) + y + 7$$

$$x = \frac{\sin(z) + y + 7}{\sqrt{3}}$$

[ 3, 3 marks ]

#### Answer 6

$$6(4x + p) = 13$$

$$24x + 6p = 13$$

$$24x = 13 - 6p$$

$$x = \frac{13 - 6p}{24}$$

#### Answer 7

$$\frac{x}{4} + 12a = 7c$$

$$\frac{x}{4} = 7c - 12a$$

$$x = 28c - 48a$$

[ 3, 3 marks ]

#### Answer 8

$$5p + 4v = \sqrt{\sin(m)}$$

$$5p = \sqrt{\sin(m)} - 4v$$

$$p = \frac{\sqrt{\sin(m)} - 4v}{5}$$

[ 3 marks ]

**Answer 9**

$$(i) \quad x + m^2 = v^2 + \frac{1}{c}$$

$$x = v^2 + \frac{1}{c} - m^2$$

$$(ii) \quad \frac{x}{8 \cos(\sqrt{k})} = f$$

$$x = 8f \cos(\sqrt{k})$$

$$(iii) \quad \frac{1}{x} = \frac{m + 3\sqrt{k}}{z + 8}$$

$$\frac{x}{1} = \frac{z + 8}{m + 3\sqrt{k}}$$

$$x = \frac{z + 8}{m + 3\sqrt{k}}$$

$$(iv) \quad \frac{1}{x} = a^2 + b^2 - 5$$

$$\frac{x}{1} = \frac{1}{a^2 + b^2 - 5}$$

$$x = \frac{1}{a^2 + b^2 - 5}$$

**[ 4 marks ]****Answer 10**

$$w - \frac{2v}{p} = m^2$$

$$w = m^2 + \frac{2v}{p}$$

**[ 2 marks ]****Answer 11**

$$\frac{4}{x} + 5k = 7m$$

$$\frac{4}{x} = 7m - 5k$$

$$\frac{x}{4} = \frac{1}{7m - 5k}$$

$$x = \frac{4}{7m - 5k}$$

**Answer 12**

$$\frac{7f}{x} + 4 = \sin(w)$$

$$\frac{7f}{x} = \sin(w) - 4$$

$$\frac{x}{7f} = \frac{1}{\sin(w) - 4}$$

$$x = \frac{7f}{\sin(w) - 4}$$

**[ 3, 3 marks ]****Answer 13**

$$\frac{1}{x + 5p} = \frac{a^2}{7 + b}$$

$$x + 5p = \frac{7 + b}{a^2}$$

$$x = \frac{7 + b}{a^2} = 5p$$

**Answer 14**

$$\frac{3 + m}{x} - 4km = u^2$$

$$\frac{3 + m}{x} = u^2 + 4km$$

$$\frac{x}{3 + m} = \frac{1}{u^2 + 4km}$$

$$x = \frac{3 + m}{u^2 + 4km}$$

**[ 3, 3 marks ]**

**Answer 15**

$$\frac{4}{5z^3 + x} - 3h = g - 1$$

$$\frac{4}{5z^3 + x} = g - 1 + 3h$$

$$\frac{5z^3 + x}{4} = \frac{1}{g - 1 + 3h}$$

$$5z^3 + x = \frac{4}{g - 1 + 3h}$$

$$x = \frac{4}{g - 1 + 3h} - 5z^3$$

**Answer 16**

$$\sqrt{x + 1} = z + 3$$

$$x + 1 = (z + 3)^2$$

$$x = (z + 3)^2 - 1$$

**[ 3, 3 marks ]****Answer 17**

$$(x - 5)^2 = m$$

$$x - 5 = \pm \sqrt{m}$$

$$x = 5 \pm \sqrt{m}$$

**Answer 18**

$$\frac{1}{\sqrt{x}} = \frac{3}{8}$$

$$\frac{\sqrt{x}}{1} = \frac{8}{3}$$

$$\sqrt{x} = \frac{8}{3}$$

square both sides

$$x = \frac{64}{9}$$

**[ 3, 3 marks ]**

## Lesson 5

### GCSE Mathematics Algebra, Rearranging Formulae

#### 5.1 $-x$



#### Example

We begin by looking at a couple of ways of answering the following question.

**Question:** Make  $x$  the subject of  $a - 4mx - 5 = 8k$

**Method 1 solution:**  $a - 4mx - 5 = 8k$

**Method 2 solution:**  $a - 4mx - 5 = 8k$

## Exercise 5.2

### Question 1

*Showing full working*, by first multiplying both sides by  $-1$ , make  $x$  the subject

$$-3x + y^3 = 17d$$

### Question 2

*Showing full working*, by first multiplying both sides by  $5$ , make  $x$  the subject

$$\frac{8w - x}{5} = h$$

**Question 3**

**Showing full working**, by first swapping the LHS and RHS, make  $x$  the subject

$$1 + w = 4x - z^2$$

**Question 4**

**Showing full working**, by first multiplying both sides by  $-1$ , make  $x$  the subject

$$p - \frac{3}{x} = E z$$

**Question 5**

**Showing full working**, by first taking the reciprocal of both sides, make  $x$  the subject

$$\frac{1}{4m - x} = \frac{k + 4}{12}$$

**Question 6**

**Showing full working**, by first swapping the LHS and RHS, make  $x$  the subject

$$w = \frac{4 + z}{x} + v^2$$

**Question 7**

**Showing full working**, by first expanding the brackets, make  $x$  the subject

$$5\left(1 - \frac{3z}{x}\right) = -k$$

**Question 8**

**Showing full working**, by first swapping the LHS and RHS, make  $x$  the subject

$$7k + 2w = \frac{x}{3}$$

Write your answer without brackets.

**Question 9**

**Showing full working**, by first subtracting  $7w$  from both sides, make  $x$  the subject

$$\frac{px}{5} + 7w = 2v$$

Write your answer without brackets.

**Question 10**

**Showing full working**, by first multiplying both sides by  $-1$ , make  $x$  the subject

$$-x + \frac{m^2}{7b + w} = \sqrt{p}$$

### 5.3 Homework

#### GCSE Mathematics Algebra, Rearranging Formulae

##### Question 1

*Showing full working*, make  $x$  the subject

$$kx - 12p = 5k$$

##### Question 2

*Showing full working*, by first multiplying both sides by  $-1$ , make  $x$  the subject

$$g - x = 7 - \frac{3k}{z^2}$$

**Question 3**

**Showing full working**, by first expanding the brackets, make  $x$  the subject

$$6(3x + 2y) = z^2 + 1$$

**Question 4**

**Showing full working**, by first taking the reciprocal of both sides, make  $x$  the subject

$$\frac{w}{hx} = \frac{1}{k}$$

**Question 5**

*Showing full working*, make  $x$  the subject

$$b - a = 6 - \frac{1}{x}$$

**Question 6**

*Showing full working*, by first swapping the LHS and RHS, make  $x$  the subject

$$7 = \frac{x}{z^2 + 4m + 3s}$$

Write your answer without brackets.

**Question 7**

**Showing full working**, by first adding  $5zx$  to both sides, make  $x$  the subject

$$s = \sqrt{p} - 5zx$$

**Question 8**

**Showing full working**, by first expanding the brackets, make  $x$  the subject

$$m\left(\frac{x}{4} - 3p\right) = f$$

**Question 9**

*Showing full working*, by first swapping the LHS and RHS, make  $x$  the subject

$$3 + 2h = \frac{x + m}{9}$$

Write your answer without brackets.

**Question 10**

*Showing full working*, make  $x$  the subject

$$u = \frac{1}{4 - x}$$

## 5.4 Answers

### GCSE Mathematics Algebra, Rearranging Formulae

#### 5.4.1 Solution to Introductory Example

##### Method 1

|               |                             |               |
|---------------|-----------------------------|---------------|
|               | $a - 4mx - 5 = 8k$          |               |
| $\times (-1)$ |                             | $\times (-1)$ |
|               | $-a + 4mx + 5 = -8k$        |               |
| $+a$          |                             | $+a$          |
|               | $4mx + 5 = a - 8k$          |               |
| $-5$          |                             | $-5$          |
|               | $4mx = a - 8k - 5$          |               |
| $\div 4m$     |                             | $\div 4m$     |
|               | $x = \frac{a - 8k - 5}{4m}$ |               |

##### Method 2

|                   |                             |                   |
|-------------------|-----------------------------|-------------------|
|                   | $a - 4mx - 5 = 8k$          |                   |
| $+4mx$            |                             | $+4mx$            |
|                   | $a - 5 = 4mx + 8k$          |                   |
| $\leftrightarrow$ |                             | $\leftrightarrow$ |
|                   | $4mx + 8k = a - 5$          |                   |
| $-8k$             |                             | $-8k$             |
|                   | $4mx = a - 8k - 5$          |                   |
| $\div 4m$         |                             | $\div 4m$         |
|                   | $x = \frac{a - 8k - 5}{4m}$ |                   |

### 5.4.2 Solutions to 5.2 Exercise

#### Answer 1

$$x = \frac{y^3 - 17d}{3}$$

#### Answer 2

$$x = 8w - 5h$$

#### Answer 3

$$x = \frac{1 + w + z^2}{4}$$

#### Answer 4

$$x = \frac{3}{p - Ez}$$

#### Answer 5

$$x = 4m - \frac{12}{k + 4}$$

#### Answer 6

$$x = \frac{4 + z}{w - v^2}$$

#### Answer 7

$$x = \frac{15z}{k + 5}$$

#### Answer 8

$$x = 21k + 6w$$

#### Answer 9

$$x = \frac{10v - 35w}{p}$$

#### Answer 10

$$x = \frac{m^2}{7b + w} - \sqrt{p}$$

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### 5.4.3 Solutions to 5.3 Homework

## GCSE Mathematics Algebra, Rearranging Formulae

#### Answer 1

$$x = \frac{5k + 12p}{k}$$

#### Answer 2

$$x = g - 7 + \frac{3k}{z^2}$$

#### Answer 3

$$x = \frac{z^2 + 1 - 12y}{18}$$

#### Answer 4

$$x = \frac{kw}{h}$$

#### Answer 5

$$x = \frac{1}{6 - b + a}$$

#### Answer 6

$$x = 7z^2 + 28m + 21s$$

#### Answer 7

$$x = \frac{\sqrt{p} - s}{5z}$$

#### Answer 8

$$x = \frac{4f + 12pm}{m}$$

#### Answer 9

$$x = 27 + 18h - m$$

#### Answer 10

$$x = 4 - \frac{1}{u}$$

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## 6.1 Squares and Square Roots

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*In algebra, there is one golden rule for manipulating all equations*  
**DO THE SAME TO THE WHOLE OF BOTH SIDES**

---

The list of actions that can be done to both sides includes

- Square both sides
- Square root both sides

## 6.2 Square both sides Example

*Showing full working*, make  $x$  the subject of the formula

$$\frac{\sqrt{x+3}}{4} = g$$

## 6.3 Square Root both side Example

*Showing full working*, make  $x$  the subject of the formula

$$\frac{x^2 + k}{2m} = f$$

## 6.4 Exercise

### Question 1

*Showing full working*, make  $x$  the subject of the formula

$$\sqrt{x + w} = 5p$$

### Question 2

*Showing full working*, make  $x$  the subject of the formula

$$x^2 + u = 2h$$

**Question 3**

*Showing full working*, make  $x$  the subject of the formula

$$\frac{1}{\sqrt{x}} = \frac{s}{3}$$

**Question 4**

*Showing full working*, make  $x$  the subject of the formula

$$\frac{1}{x^2} + 7f = h + k$$

**Question 5**

*Showing full working*, make  $x$  the subject of the formula

$$\frac{\sqrt{x+5}}{m} = 7$$

**Question 6**

*Showing full working*, make  $x$  the subject of the formula

$$\frac{x^2 - 7}{8d} = f$$

**Question 7**

*Showing full working*, make  $x$  the subject of the formula

$$7m - \sqrt{x} = 0$$

**Question 8**

*Showing full working*, make  $x$  the subject of the formula

$$\sqrt{\frac{c+p}{x}} = 4$$

**Question 9**

*Showing full working*, make  $x$  the subject of the formula

$$\frac{3m x^2}{p} = k$$

**Question 10**

*Showing full working*, make  $x$  the subject of the formula

$$\frac{\sqrt{x^2 + 5}}{w} = 6$$

**Question 11**

*Showing full working*, make  $x$  the subject of the formula

$$\frac{w}{g} = \frac{x^2 + m}{3 + s}$$

**Question 12**

*Showing full working*, make  $x$  the subject of the formula

$$\frac{3}{\sqrt{x + 5}} = \frac{z}{4t}$$

## 6.5 Quick 'In your head' Questions

### GCSE Mathematics Algebra, Rearranging Formulae

There are TWO operations to be done to both sides to get from question to answer.  
Try to do both in your head.  
Do not write down any working.

#### Question 1

Make  $x$  the subject;  $\sqrt{x + h} = k$

#### Question 2

Make  $x$  the subject;  $x^2 - 6 = g$

#### Question 3

Make  $x$  the subject;  $2d = \frac{x}{3m}$

#### Question 4

Make  $x$  the subject

$$\frac{1}{x} + k = 4w$$

#### Question 5

Make  $x$  the subject;  $5x^2 = p^{11} + 2$

**Question 6**

Make  $x$  the subject;  $\sqrt{\frac{x}{g}} = 10s$

**Question 7**

Make  $x$  the subject;  $8 - x^2 = c$

**Question 8**

Make  $x$  the subject;  $x^2(k^2 + p) = v$

**Question 9**

Make  $x$  the subject;  $\frac{\sqrt{x}}{e} = \frac{5}{7}$

**Question 10**

Make  $x$  the subject;  $\frac{1}{\sqrt{x}} = 9n$

## 6.6 Answers

### GCSE Mathematics Algebra, Rearranging Formulae

#### 6.6.1 Solutions to Square both sides Example 6.2

$$x = 16g^2 - 3$$

#### 6.6.2 Solutions to Square Root both sides Example 6.3

$$x = \pm \sqrt{2mf - k}$$

#### 6.6.3 Solutions to Exercise 6.4

##### Answer 1

$$x = 25p^2 - w$$

##### Answer 2

$$x = \pm \sqrt{2h - u}$$

##### Answer 3

$$x = \frac{9}{s^2}$$

##### Answer 4

$$x = \pm \sqrt{\frac{1}{h + k - 7f}}$$

##### Answer 5

$$x = 49m^2 - 5$$

##### Answer 6

$$x = \pm \sqrt{8df + 7}$$

##### Answer 7

$$x = 49m^2$$

##### Answer 8

$$x = \frac{c + p}{16}$$

##### Answer 9

$$x = \pm \sqrt{\frac{kp}{3m}}$$

##### Answer 10

$$x = \pm \sqrt{36w^2 - 5}$$

##### Answer 11

$$x = \pm \sqrt{\frac{w(3 + s)}{g} - m}$$

##### Answer 12

$$x = \frac{144t^2}{z^2} - 5$$

### 6.6.4 Solutions to Quick 'In your head' Questions 6.5

#### GCSE Mathematics Algebra, Rearranging Formulae

##### Answer 1

$$x = k^2 - h$$

##### Answer 3

$$x = 6dm$$

##### Answer 5

$$x = \pm \sqrt{\frac{p^{11} + 2}{5}}$$

##### Answer 7

$$x = \pm \sqrt{8 - c}$$

##### Answer 9

$$x = \frac{25e^2}{49}$$

##### Answer 2

$$x = \pm \sqrt{g + 6}$$

##### Answer 4

$$x = \frac{1}{4w - k}$$

##### Answer 6

$$x = 100s^2g$$

##### Answer 8

$$x = \pm \sqrt{\frac{v}{k^2 + p}}$$

##### Answer 10

$$x = \frac{1}{81n^2}$$

## Lesson 7

### GCSE Mathematics Algebra, Rearranging Formulae

#### 7.1 Making letters, other than $x$ , the subject

##### Example 1

*Showing full working*, make  $k$  the subject

$$\frac{3m + 10p - 8a^2}{k} = \frac{1}{6}$$

##### Example 2

*Showing full working*, make  $m$  the subject

$$\sqrt{\frac{4 + h}{m}} = \frac{p}{5}$$

## 7.2 Exercise

### Question 1

*Showing full working*, make  $a$  the subject

$$\sqrt{\frac{a}{3}} = 5w$$

### Question 2

*Showing full working*, make  $u$  the subject

$$\frac{3}{v + 4u} = \frac{1}{5e^2 + 7s}$$

**Question 3**

*Showing full working*, make  $w$  the subject

$$\frac{2.4}{s} = w^2 - m$$

**Question 4**

*Showing full working*, make  $m$  the subject

$$y = mx + c$$

**Question 5**

*Showing full working*, make  $c$  the subject

$$\sqrt{c^2 - 5k} = 3p$$

**Question 6**

*Showing full working*, make  $b$  the subject

$$\frac{b}{8} - 4h = 2t + 5$$

Write your answer without any brackets.

**Question 7**

*Showing full working*, make  $f$  the subject

$$\sqrt{3g + f} = \frac{3v}{w}$$

**Question 8**

*Showing full working*, by first multiplying both sides by 3, make  $z$  the subject

$$\frac{1}{3z^2} = 2r + y$$

Write your answer without any brackets

**Question 9**

*Showing full working*, by first expanding the brackets, make  $w$  the subject

$$h = 3 \left( \frac{4}{w} + 5 \right)$$

**Question 10**

*Showing full working*, make  $p$  the subject

$$\frac{1}{5p^2} = \frac{3a + 2b}{16c^2}$$

### 7.3 Solutions to Exercise 7.2

#### GCSE Mathematics Algebra, Rearranging Formulae

##### Answer 1

$$a = 75w^2$$

##### Answer 3

$$w = \pm \sqrt{\frac{2.4}{s} + m}$$

##### Answer 5

$$c = \pm \sqrt{9p^2 + 5k}$$

##### Answer 7

$$f = \frac{9v^2}{w^2} - 3g$$

##### Answer 9

$$w = \frac{12}{h - 15}$$

##### Answer 2

$$u = \frac{15e^2 + 21s - v}{4}$$

##### Answer 4

$$m = \frac{y - c}{x}$$

##### Answer 6

$$b = 16t + 40 + 32h$$

##### Answer 8

$$z = \pm \sqrt{\frac{1}{6r + 3y}}$$

$$\text{or } z = \pm \frac{1}{\sqrt{6r + 3y}}$$

##### Answer 10

$$p = \pm \frac{4c}{\sqrt{15a + 10b}}$$

## Lesson 8

### GCSE Mathematics Algebra, Rearranging Formulae

#### 8.1 Isolation techniques



Photograph by Simon Wiffen

The GCSE examination often has a *Change of Subject* question in which the variable to be made the subject occurs more than once.

#### 8.2 An Easier Example

Make  $x$  the subject;  $2h + 3x = k^2 - 8x$

#### 8.3 A Harder Example

*Showing full working*, make  $x$  the subject

$$2y = \frac{3 + 7x}{4 + 5x}$$

## 8.4 Exercise

### Question 1

*Showing full working*, make  $x$  the subject

$$7m + 9x = 4x + b$$

### Question 2

*Showing full working*, by first expanding the brackets, make  $x$  the subject

$$6(x + 4y) = 5(7c - 3x)$$

**Question 3**

*Showing full working*, by first making brackets, make  $x$  the subject

$$11x + mx = k^2$$

**Question 4**

*Showing full working*, make  $x$  the subject

$$\frac{1}{6x - y} = \frac{1}{13k - 5x}$$

**Question 5**

**Showing full working,** by first multiplying both sides by  $x$ , make  $x$  the subject

$$19 = \frac{6(3m + x + k^2)}{x}$$

**Question 6**

**Showing full working,** by first adding  $5x^2$  to both sides, make  $x$  the subject

$$8x^2 + m = w - 5x^2$$

**Question 7**

*Showing full working*, make  $x$  the subject

$$4(y + 3x - 5) = x + 17y$$

**Question 8**

*Showing full working*, by first expanding the brackets, make  $x$  the subject

$$3(4z^2 - 9x) = 7(2x + 7k)$$

**Question 9**

*Showing full working*, make  $x$  the subject

$$y = \sqrt{\frac{3x + 1}{x - 1}}$$

**Question 10**

*Showing full working*, make  $x$  the subject

$$9 = \frac{4x + e^2}{3x + 2wh}$$

## 8.4 Solutions to Exercise 8.3

### GCSE Mathematics Algebra, Rearranging Formulae

#### Answer 1

$$x = \frac{b - 7m}{5}$$

#### Answer 2

$$x = \frac{35c - 24y}{21}$$

#### Answer 3

$$x = \frac{k^2}{11 + m}$$

#### Answer 4

$$x = \frac{13k + y}{11}$$

#### Answer 5

$$x = \frac{18m + 6k^2}{13}$$

#### Answer 6

$$x = \pm \sqrt{\frac{w - m}{13}}$$

#### Answer 7

$$x = \frac{13y + 20}{11}$$

#### Answer 8

$$x = \frac{12z^2 - 49k}{41}$$

#### Answer 9

$$x = \frac{y^2 + 1}{y^2 - 3}$$

#### Answer 10

$$x = \frac{e^2 - 18wh}{23}$$

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## Lesson 9

### GCSE Mathematics Algebra, Rearranging Formulae

#### 9.1 Test

##### Question 1

If I divide one side of an equation by 8, what must I do to the other side ?

[ 1 mark ]

##### Question 2

Quick 'In your head' Questions.

In each of the following, there is one manipulation that takes you from the question to the answer and so there is no need for any working.

( i )      Make  $x$  the subject;       $x - 7h^2 = 4w$

[ 2 marks ]

( ii )      Make  $x$  the subject;       $15gx = k$

[ 2 marks ]

( iii )      Make  $x$  the subject;       $\frac{x}{9} = \sqrt{f}$

[ 2 marks ]

( iv )      Make  $x$  the subject;       $\frac{1}{x} = \frac{m + 2s}{8v^2}$

[ 2 marks ]

**Question 3**

*Showing full working*, make x the subject

$$a + xk^2 = 13 + s^2$$

[ 3 marks ]

**Question 4**

*Showing full working*, by first expanding the brackets, make x the subject

$$5 ( 7x - 9y ) = 1 - u^2$$

[ 3 marks ]

**Question 5**

*GCSE examination question from November 2004, paper 4H, Q15*

Make  $v$  the subject of the formula

$$m(v - u) = I$$

[ 3 marks ]

**Question 6**

*Showing full working*, make  $x$  the subject

$$11g + 13x = 8x + 3b$$

[ 3 marks ]

**Question 7**

*GCSE examination question from November 2007, paper 3H, Q11*

Make  $x$  the subject of the formula

$$3x - y = x + 7$$

**[ 3 marks ]**

**Question 8**

*GCSE examination question from January 2012, paper 3H, Q11*

Make  $y$  the subject of the formula

$$3 ( y + 2x - 1 ) = x + 5y$$

**[ 3 marks ]**

**Question 9**

*Showing full working*, by first making brackets, make  $x$  the subject

$$17x + wx = p^2$$

[ 3 marks ]

**Question 10**

*Showing full working*, make  $x$  the subject

$$\frac{1}{8x - 5y} = \frac{1}{13h - 5x}$$

[ 3 marks ]

**Question 11**

*GCSE examination question from November 2006, paper 4H, Q4*

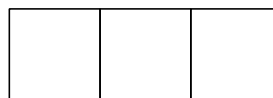
Here is a pattern of shapes made from centimetre squares.



**Shape  
number 1**



**Shape  
number 2**



**Shape  
number 3**

This rule can be used to find the perimeter of a shape in this pattern.

---

Add 1 to the Shape number and then multiply your answer by 2

---

$P$  cm is the perimeter of Shape number  $n$ .

**( a )** Write down a formula for  $P$  in terms of  $n$ .

**[ 3 marks ]**

**( b )** Make  $n$  the subject of the formula in part **( a )**.

**[ 3 marks ]**

**Question 12**

*Showing full working*, by first squaring both sides, make  $x$  the subject

$$\sqrt{14x^2 + v - 3c^2} = 5x$$

[ 4 marks ]

**Question 13**

*GCSE examination question from January 2014, paper 3H, Q16*

Given that  $y$  is positive, make  $y$  the subject of

$$y = \sqrt{ay^2 + n}$$

Show clear algebraic working.

[ 5 marks ]

**Question 14**

*GCSE examination question from November 2010, paper 3H, Q18*

Make  $x$  the subject of

$$P = \frac{100 (y - x)}{x}$$

[ 4 marks ]

**Question 15**

*GCSE examination question from November 2009, paper 3H, Q11*

Make  $a$  the subject of

$$P = \sqrt{ab}$$

[ 2 marks ]

**Question 16**

*GCSE examination question from May 2012, paper 4H, Q10*

Given that  $r$  is positive, make  $r$  the subject of the formula

$$V = \pi r^2 h$$

[ 2 marks ]

**Question 17**

*GCSE examination question from January 2013, paper 3H, Q12*

Make  $h$  the subject of the formula

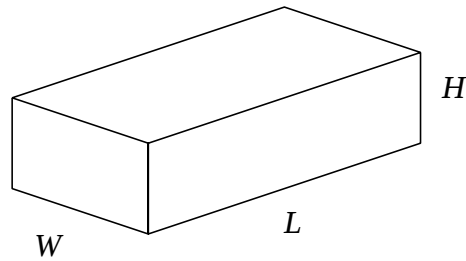
$$A = 2\pi r (r + h)$$

[ 2 marks ]

**Question 18**

*GCSE examination question from May 2009, paper 3H, Q13*

Here is a cuboid with length  $L$ , width  $W$  and height  $H$ .



The total surface area,  $A$ , of the cuboid is given by the formula

$$A = 2 ( LW + HW + HL )$$

- ( a )     $A = 70$     $W = 4$     $H = 2$   
Work out the value of  $L$

[ 3 marks ]

- ( b )    Make  $W$  the subject of the formula

$$A = 2 ( LW + HW + HL )$$

[ 4 marks ]

**Question 19**

*GCSE examination question from May 2014, paper 3H, Q17*

Make  $x$  the subject of;

$$y = \sqrt{\frac{2x + 1}{x - 1}}$$

[ 4 marks ]

**Question 20**

*GCSE examination question from June 2011, paper 4H, Q15 ( c )*

Make  $q$  the subject of the formula

$$p = \sqrt{q} - 5r$$

[ 2 marks ]

**Question 21**

*GCSE examination question from May 2006, paper 3H, Q20*

Make  $R$  the subject of the formula

$$A = \pi (R + r) (R - r)$$

**[ 4 marks ]**

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Teachers may obtain detailed worked solutions to the exercises by email from [MHHShrewsbury@Gmail.com](mailto:MHHShrewsbury@Gmail.com)

## 9.2 Answers

## GCSE Mathematics Algebra, Rearranging Formulae

### Answer 1

Divide the other side by 8

### Answer 2

( i )

$$x = 4w + 7h^2$$

( iii )

$$x = 9\sqrt{f}$$

### Answer 3

$$x = \frac{13 + s^2 - a}{k^2}$$

### Answer 5

$$v = \frac{I + um}{m}$$

### Answer 7

$$x = \frac{7 + y}{2}$$

### Answer 9

$$x = \frac{p^2}{(17 + w)}$$

### Answer 11

( a )

$$P = 2(n + 1) \quad \text{or} \quad P = 2n + 2$$

( b )

$$n = \frac{P}{2} - 1 \quad \text{or} \quad n = \frac{P - 2}{2}$$

### Answer 12

$$x = \pm \sqrt{\frac{v - 3c^2}{11}}$$

( ii )

$$x = \frac{k}{15g}$$

( iv )

$$x = \frac{8v^2}{m + 2s}$$

### Answer 4

$$x = \frac{1 - u^2 + 45y}{35}$$

### Answer 6

$$x = \frac{3b - 11g}{5}$$

### Answer 8

$$y = \frac{5x - 3}{2}$$

### Answer 10

$$x = \frac{13h + 5y}{13}$$

### Answer 13

$$y = \sqrt{\frac{n}{1 - a}}$$

(Told y is positive )

**Answer 14**

$$x = \frac{100y}{(P + 100)}$$

**Answer 15**

$$a = \frac{p^2}{b}$$

**Answer 16**

$$r = \sqrt{\frac{V}{\pi h}}$$

**Answer 17**

$$h = \frac{A - 2\pi r^2}{2\pi r}$$

$$\text{or } h = \frac{A}{2\pi r} - r$$

**Answer 18****( a )**

$$L = 4.5$$

**( b )**

$$W = \frac{\frac{A}{2} - HL}{(L + H)} \quad \text{for example}$$

**Answer 19**

$$x = \frac{1 + y^2}{y^2 - 2}$$

**Answer 20**

$$q = (p + 5r)^2$$

**Answer 21**

$$R = \pm \sqrt{\frac{A}{\pi} + r^2}$$

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