

1.4 Answers

1.4.1 Solutions (To 1.2 Example)

(i)

$$\begin{aligned}(2 + 3x)^4 &= (1) \times (2)^4 \times (3x)^0 \\ &\quad + (4) \times (2)^3 \times (3x)^1 \\ &\quad + (6) \times (2)^2 \times (3x)^2 \\ &\quad + (4) \times (2)^1 \times (3x)^3 \\ &\quad + (1) \times (2)^0 \times (3x)^4 \\ &= 16 + 96x + 216x^2 + 216x^3 + 81x^4\end{aligned}$$

(ii)

$$(2 + 3\sqrt{x})^4 = 16 + 96\sqrt{x} + 216x + 216(\sqrt{x})^3 + 81x^2$$

1.4.2 Solutions (To 1.3 Exercise)

Answer 1

(i)

$$\begin{aligned}(5 - 2x)^3 &= (1) \times (5)^3 \times (-2x)^0 \\ &\quad + (3) \times (5)^2 \times (-2x)^1 \\ &\quad + (3) \times (5)^1 \times (-2x)^2 \\ &\quad + (1) \times (5)^0 \times (-2x)^3 \\ &= 125 - 150x + 60x^2 - 8x^3\end{aligned}$$

(ii)

$$= 125 - 150\sqrt{x} + 60x - 8(\sqrt{x})^3$$

Answer 2

(i)

$$\begin{aligned}(11 - x)^4 &= (1) \times (11)^4 \times (-x)^0 \\ &\quad + (4) \times (11)^3 \times (-x)^1 \\ &\quad + (6) \times (11)^2 \times (-x)^2 \\ &\quad + (4) \times (11)^1 \times (-x)^3 \\ &\quad + (1) \times (11)^0 \times (-x)^4 \\ &= 14641 - 5324x + 726x^2 - 44x^3 + x^4\end{aligned}$$

(ii)

$$\begin{aligned}(10 + x^4)^5 &= (1) \times (10)^5 \times (x^4)^0 \\&\quad + (5) \times (10)^4 \times (x^4)^1 \\&\quad + (10) \times (10)^3 \times (x^4)^2 \\&\quad + (10) \times (10)^2 \times (x^4)^3 \\&\quad + (5) \times (10)^1 \times (x^4)^4 \\&\quad + (1) \times (10)^0 \times (x^4)^5 \\&= 100\,000 + 50\,000x^4 + 10\,000x^8 + 1\,000x^{12} + 50x^{16} + x^{20}\end{aligned}$$

Answer 3

(i)

$$\begin{aligned}\left(x + \frac{5}{x}\right)^4 &= (1) \times (x)^4 \times \left(\frac{5}{x}\right)^0 \\&\quad + (4) \times (x)^3 \times \left(\frac{5}{x}\right)^1 \\&\quad + (6) \times (x)^2 \times \left(\frac{5}{x}\right)^2 \\&\quad + (4) \times (x)^1 \times \left(\frac{5}{x}\right)^3 \\&\quad + (1) \times (x)^0 \times \left(\frac{5}{x}\right)^4 \\&= x^4 + 20x^2 + 150 + \frac{500}{x^2} + \frac{625}{x^4}\end{aligned}$$

(ii) 150 or "3rd term"

(iii) 20

Answer 4**(i)**

$$\begin{aligned}\left(1 + \frac{x}{2}\right)^4 &= (1) \times (1)^4 \times \left(\frac{x}{2}\right)^0 \\ &\quad + (4) \times (1)^3 \times \left(\frac{x}{2}\right)^1 \\ &\quad + (6) \times (1)^2 \times \left(\frac{x}{2}\right)^2 \\ &\quad + (4) \times (1)^1 \times \left(\frac{x}{2}\right)^3 \\ &\quad + (1) \times (1)^0 \times \left(\frac{x}{2}\right)^4 \\ &= 1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4\end{aligned}$$

(ii)

$$\begin{aligned}(4 + 2x)^4 &= [(4 + 2x)]^4 \\ &= \left[4 \left(1 + \frac{x}{2}\right) \right]^4 \\ &= 4^4 \left(1 + \frac{x}{2}\right)^4 \\ &= 256 \left(1 + \frac{x}{2}\right)^4\end{aligned}$$

(iii)

$$\begin{aligned}(4 + 2x)^4 &= 256 \left(1 + \frac{x}{2}\right)^4 \\ &= 256 \left(1 + 2x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4\right) \\ &= 256 + 512x + 384x^2 + 128x^3 + 16x^4\end{aligned}$$

Answer 5**(i)**

$$\begin{aligned}
(6 + 9x^2)^5 &= [(6 + 9x^2)]^5 \\
&= \left[6 \left(1 + \frac{3}{2}x^2 \right) \right]^5 \\
&= 6^5 \left(1 + \frac{3}{2}x^2 \right)^5 \\
&= 7776 \left(1 + \frac{3}{2}x^2 \right)^5 \\
\therefore k &= 7776 \quad \text{and} \quad a = \frac{3}{2}
\end{aligned}$$

(ii)

$$\begin{aligned}
\left(1 + \frac{3}{2}x^2 \right)^5 &= (1) \times (1)^5 \times \left(\frac{3}{2}x^2 \right)^0 \\
&\quad + (5) \times (1)^4 \times \left(\frac{3}{2}x^2 \right)^1 \\
&\quad + (10) \times (1)^3 \times \left(\frac{3}{2}x^2 \right)^2 \\
&\quad + (10) \times (1)^2 \times \left(\frac{3}{2}x^2 \right)^3 \\
&\quad + (5) \times (1)^1 \times \left(\frac{3}{2}x^2 \right)^4 \\
&\quad + (1) \times (1)^0 \times \left(\frac{3}{2}x^2 \right)^5 \\
&= 1 + \frac{15}{2}x^2 + \frac{45}{2}x^4 + \frac{135}{4}x^6 + \frac{405}{16}x^8 + \frac{243}{32}x^{10}
\end{aligned}$$

(iii)

$$\begin{aligned}
(6 + 9x^2)^5 &= 7776 \left(1 + \frac{3}{2}x^2 \right)^5 \\
&= 7776 \left(1 + \frac{15}{2}x^2 + \frac{45}{2}x^4 + \frac{135}{4}x^6 + \frac{405}{16}x^8 + \frac{243}{32}x^{10} \right) \\
&= 7776 + 58320x^2 + 174960x^4 + 262440x^6 + 196830x^8 + 59049x^{10}
\end{aligned}$$

Answer 6

$$\begin{aligned}\left(\frac{1}{3x^2} + x^3\right)^4 &= (1) \times \left(\frac{1}{3x^2}\right)^4 \times (x^3)^0 \\ &\quad + (4) \times \left(\frac{1}{3x^2}\right)^3 \times (x^3)^1 \\ &\quad + (6) \times \left(\frac{1}{3x^2}\right)^2 \times (x^3)^2 \\ &\quad + (4) \times \left(\frac{1}{3x^2}\right)^1 \times (x^3)^3 \\ &\quad + (1) \times \left(\frac{1}{3x^2}\right)^0 \times (x^3)^4 \\ &= \frac{1}{81x^8} + \frac{4}{27x^3} + \frac{2}{3}x^2 + \frac{4}{3}x^7 + x^{12}\end{aligned}$$