2.4 Answers

2.4.1 Solutions (To 2.2 Example)

$$Using \qquad {}^{n}C_{r} = \frac{n\,!}{r\,!\;(n-r)\,!}$$

$$we\ get \qquad {}^{a+6}C_{a+4} = \frac{(a+6)\,!}{(a+4)\,!\;((a+6)-(a+4))\,!}$$

$$= \frac{(a+6)\,(a+5)\,(a+4)!}{(a+4)\,!\;(a+6\;-\;a-4))\,!}$$

$$= \frac{(a+6)\,(a+5)}{2}$$

2.4.2 Solutions (To 2.3 Exercise)

Answer 1

(i) 5040 **(ii)** 1

Answer 2

(i)

$$\frac{100\,!}{98\,!} = \frac{100 \times 99 \times 98\,!}{98\,!} = 9900$$

(ii)

$$\frac{100\,!}{2\,!\;(100-2)\,!} = \frac{100\,!}{2\,!\;(98)\,!} = \frac{1}{2!} \times 9900 = 4950$$

Answer 3

$$Using \qquad {}^{n}C_{r} = \frac{n\,!}{r\,!\;(n-r)\,!}$$

$$we\ get \qquad {}^{a+11}C_{a+9} = \frac{(a+11)\,!}{(a+9)\,!\;((a+11)-(a+9))\,!}$$

$$= \frac{(a+11)\,(a+10)\,(a+9)!}{(a+9)\,!\;(a+11\;-\;a-9))\,!}$$

$$= \frac{(a+11)\,(a+10)}{2}$$

Answer 4

(i)

$$\binom{11}{7} = 330$$

(ii)

$${}^{14}C_5 = 2002$$

Answer 5

$$\begin{aligned}(a + bx)^n = {} & \binom{n}{0} \times a^n \times (bx)^0 \\ & + \binom{n}{1} \times a^{n-1} \times (bx)^1 \\ & + \binom{n}{2} \times a^{n-2} \times (bx)^2 \\ & + \binom{n}{3} \times a^{n-3} \times (bx)^3 \\ & + \ldots\end{aligned}$$

becomes....

$$\begin{aligned}(1 + x)^n = {} & \frac{n!}{0!\,(n-0)!} \times 1^n \times (x)^0 \\ & + \frac{n!}{1!\,(n-1)!} \times 1^{n-1} \times (x)^1 \\ & + \frac{n!}{2!\,(n-2)!} \times 1^{n-2} \times (x)^2 \\ & + \frac{n!}{3!\,(n-3)!} \times 1^{n-3} \times (x)^3 \\ & + \ldots\end{aligned}$$

which simplifies to...

$$\begin{aligned}(1 + x)^n = {} & 1 \\ & + n\,x \\ & + \frac{n\,(n-1)}{2!}\,x^2 \\ & + \frac{n\,(n-1)\,(n-2)}{3!}\,x^3 \\ & + \ldots\end{aligned}$$

Answer 6

$$\frac{1}{(1 + 6x)} = (1 + 6x)^{-1}$$

The Binomial Theorem

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

$$\textit{... with } n = -1 \textit{ and } x \textit{ replaced with } 6x \textit{ yields...}$$

$$(1 + 6x)^{-1} = 1 + (-1)(6x) + \frac{(-1)(-2)}{2}(6x)^2 + \frac{(-1)(-2)(-3)}{6}(6x)^3$$

$$= 1 - 6x + 36x^2 - 216x^3 + \ldots$$

$$\textit{valid for } |x| < \frac{1}{6}$$

Answer 7

$$\sqrt{1 + 3x} = (1 + 3x)^{\frac{1}{2}}$$

The Binomial Theorem

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

$$\textit{... with } n = \frac{1}{2} \textit{ and } x \textit{ replaced with } 3x \textit{ yields...}$$

$$(1 + 3x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(3x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(3x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}(3x)^3$$

$$= 1 + \frac{3}{2}x - \frac{9}{8}x^2 + \frac{27}{16}x^3 + \ldots$$

$$\textit{valid for } |x| < \frac{1}{3}$$

Answer 8

$$\frac{1}{(1 - x)^2} = (1 - x)^{-2}$$

The Binomial Theorem

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

$$\textit{... with } n = -2 \textit{ and } x \textit{ replaced with } -x \textit{ yields...}$$

$$(1 - x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)}{2}(-x)^2 + \frac{(-2)(-3)(-4)}{6}(-x)^3$$

$$= 1 + 2x + 3x^2 + 4x^3 + \ldots$$

$$\textit{valid for } |x| < 1$$

Answer 9

There is an issue in this question with the number 3 being in the position where the Binomial Theorem requires the number 1

$$\frac{1}{(3+x)} = (3+x)^{-1}$$

$$= \left[3\left(1+\frac{x}{3}\right)\right]^{-1}$$

$$= \frac{1}{3}\left(1+\frac{x}{3}\right)^{-1}$$

Ignoring the third for the time being...

The Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ \ldots$$

$$\ldots\ \textit{with}\ \ n = -1\ \ \textit{and}\ \ x\ \ \textit{replaced with}\ \frac{x}{3}\ \textit{yields}\ldots$$

$$\left(1+\frac{x}{3}\right)^{-1} = 1 + (-1)\left(\frac{x}{3}\right) + \frac{(-1)\ (-2)}{2}\left(\frac{x}{3}\right)^2 + \frac{(-1)\ (-2)\ (-3)}{6}\left(\frac{x}{3}\right)^3$$

$$= 1 - \frac{x}{3} + \frac{x^2}{9} - \frac{x^3}{27} + \ldots$$

Now, don't forget about that ignored third !

$$\frac{1}{3}\left(1+\frac{x}{3}\right)^{-1} = \frac{1}{3} - \frac{x}{9} + \frac{x^2}{27} - \frac{x^3}{81} + \ldots$$

$$\textit{valid for}\ |x| < 3$$