

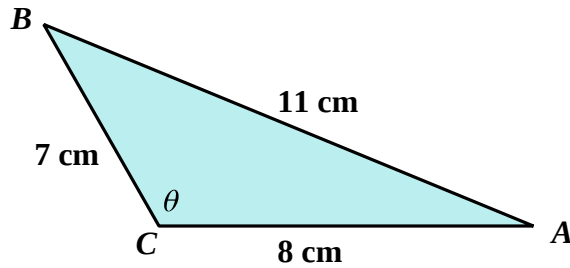
## 2.5 Answers

### A-Level Pure Mathematics : Year 2 Applications of Trigonometry

#### 2.5.1 Solutions to 2.4 Exercise

##### Answer 1

( a ) Sketch of the triangle,



$$\begin{aligned}\cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{7^2 + 8^2 - 11^2}{2 \times 7 \times 8} \\ &= -\frac{1}{14} \\ C &= 1.64^\circ\end{aligned}$$

[ 3 marks ]

( b )

$$\begin{aligned}\text{Area } \Delta &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 7 \times 8 \times \sin 1.6423 \\ &= 27.9 \text{ cm}^2\end{aligned}$$

[ 3 marks ]

**Answer 2****( a )**

$$\begin{aligned}
 \cos Q &= \frac{p^2 + r^2 - q^2}{2 p r} \\
 &= \frac{6^2 + 6^2 - (6\sqrt{3})^2}{2 \times 6 \times 6} \\
 &= -\frac{1}{2} \\
 C &= \frac{2\pi}{3} \text{ radians } \textit{*exact*}
 \end{aligned}$$

**[ 3 marks ]****( b )**

$$\begin{aligned}
 \textit{Area Sector} &= \frac{1}{2} r^2 \theta \\
 &= \frac{1}{2} \times 6^2 \times \frac{2\pi}{3} \\
 &= 12\pi \text{ m}^2 \quad \square
 \end{aligned}$$

**[ 2 marks ]****( c )**

$$\begin{aligned}
 \textit{Area } \Delta &= \frac{1}{2} ab \sin C \\
 &= \frac{1}{2} \times 6 \times 6 \times \sin\left(\frac{2\pi}{3}\right) \\
 &= 18 \times \frac{\sqrt{3}}{2} \\
 &= 9\sqrt{3} \text{ m}^2
 \end{aligned}$$

**[ 2 marks ]****( d )**

$$\begin{aligned}
 \textit{Area Segment} &= \textit{Area Sector} - \textit{Area } \Delta \\
 &= 12\pi - 9\sqrt{3} \\
 &= 22.1 \text{ m}^2
 \end{aligned}$$

**[ 2 marks ]****( e )**

$$\begin{aligned}
 \textit{Perimeter Sector} &= 6 + 6 + \textit{Arc Length} \\
 &= 12 + r\theta \\
 &= 12 + 6 \times \frac{2\pi}{3} \\
 &= 24.6 \text{ m}
 \end{aligned}$$

**[ 2 marks ]**

**Answer 3****( a )** In  $\triangle COD$ ,

$$\begin{aligned}\cos O &= \frac{d^2 + c^2 - o^2}{2 \times d \times c} \\ &= \frac{8^2 + 8^2 - 7^2}{2 \times 8 \times 8} \\ &= \frac{79}{128}\end{aligned}$$

$$O = 0.906 \text{ radians to 3 significant figures} \quad \square$$

**[ 2 marks ]****( b )**

$$\angle COB = \angle DOA = x$$

$$\therefore 2x + 0.906 = \pi \text{ as } AOB \text{ is a straight line}$$

$$x = 1.118 \text{ radians}$$

$$\text{Arc Length } BC = \text{Arc Length } AD = r\theta$$

$$8 \times 1.118 = 8.942 \text{ m}$$

$$\begin{aligned}\text{Total Perimeter} &= 16 + 8.942 + 7 + 8.942 \\ &= 40.9 \text{ cm}\end{aligned}$$

**[ 3 marks ]****( c )**

$$\text{Area Sector } COB = \text{Area Sector } DOA = \frac{1}{2} r^2 \theta$$

$$\frac{1}{2} \times 8^2 \times 1.118 = 35.776 \text{ m}^2$$

$$\text{Area } \triangle = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 8 \times 8 \times \sin 0.906$$

$$= 25.185 \text{ m}^2$$

$$\begin{aligned}\text{Total Area} &= 35.776 + 25.185 + 35.776 \\ &= 96.7 \text{ m}^2\end{aligned}$$

**[ 3 marks ]**

**Answer 4****( a )**

$$\begin{aligned} \text{Arc Length} &= r \theta \\ &= 3.5 \times 1.77 \\ &= 6.20 \text{ m} \end{aligned}$$

**[ 2 marks ]****( b )**

$$\begin{aligned} \text{Area Sector} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 3.5^2 \times 1.77 \\ &= 10.84 \text{ m}^2 \end{aligned}$$

**[ 2 marks ]****( c )**

$$\begin{aligned} \angle AFB &= \angle EFD = x \\ 2x + 1.77 &= \pi \text{ as } AE \text{ is a straight line} \\ x &= 0.6858 \\ \text{Area } \triangle AFB &= \text{Area } \triangle EFD = \frac{1}{2} ab \sin C \\ \frac{1}{2} \times 3.7 \times 3.5 \times \sin 0.6858 &= 4.101 \text{ m}^2 \\ \text{Total Area} &= 4.101 + 10.84 + 4.101 \\ &= 19.04 \text{ m}^2 \end{aligned}$$

**[ 4 marks ]**

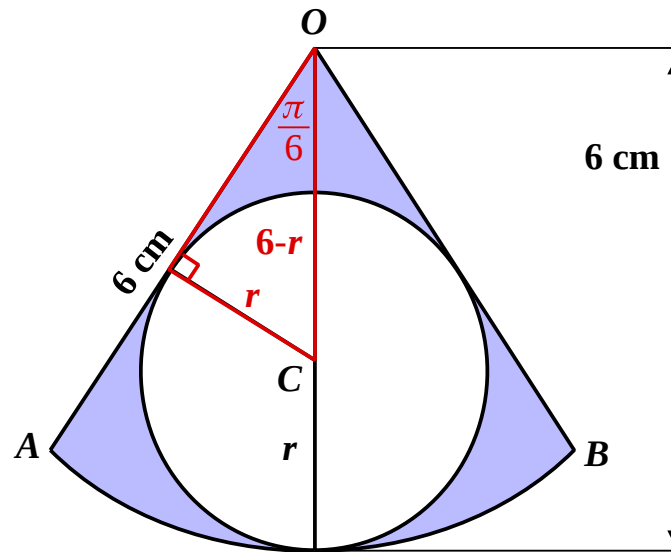
**Answer 5**

**( a )**

$$\begin{aligned} \text{Area Sector} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} \\ &= 6\pi \text{ m}^2 \quad (18.85 \text{ m}^2) \end{aligned}$$

**[ 2 marks ]**

**( b )**



$$\begin{aligned} \sin\left(\frac{\pi}{6}\right) &= \frac{r}{6-r} \\ \frac{1}{2} &= \frac{r}{6-r} \\ 6-r &= 2r \\ 3r &= 6 \\ r &= 2 \text{ cm} \end{aligned}$$

**[ 3 marks ]**

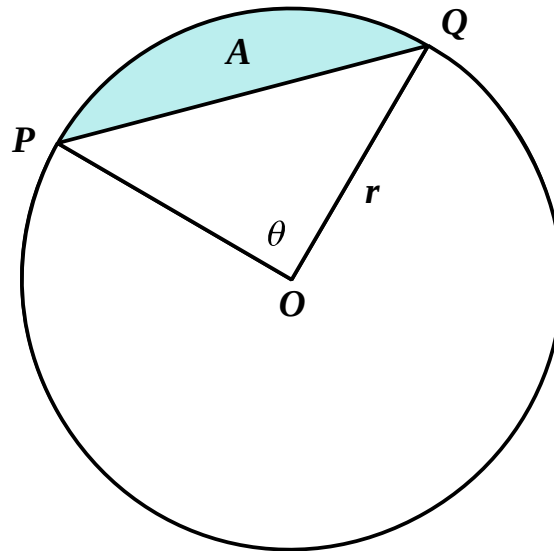
**( c )**

$$\begin{aligned} \text{Area Shaded} &= \text{Area Sector} - \text{Area Circle} \\ &= \frac{1}{2} r_s^2 \theta - \pi r_c^2 \\ &= 6\pi - \pi \times 2^2 \\ &= 6\pi - 4\pi \\ &= 2\pi \\ &= 6.28 \text{ m}^2 \end{aligned}$$

**[ 2 marks ]**

**Answer 6**

**( i )**



**[ 2 marks ]**

**( ii )**

$$\begin{aligned} \text{Area } A &= \text{Area Sector} - \text{Area } \Delta \\ &= \frac{1}{2} r^2 \theta - \frac{1}{2} \times r \times r \times \sin \theta \\ &= \frac{1}{2} r^2 ( \theta - \sin \theta ) \end{aligned}$$

**[ 3 marks ]**