3.6 Answers

3.6.1 Solutions to 3.1.3 Introductory Example

θ radians	0.1	0.2	0.3	0.4	0.5
θ degrees	5.7	11.5	17.2	22.9	28.6

[5 marks]

3.6.2 Solutions to 3.2 Investigation

θ^c	0.1	0.2	0.3	0.4	0.5
$\sin\theta$	0.100	0.199	0.296	0.389	0.479
$\cos\theta$	0.995	0.980	0.955	0.921	0.878
$\tan\theta$	0.100	0.203	0.309	0.423	0.546
$1 - \frac{\theta^2}{2}$	0.995	0.980	0.955	0.920	0.875

[5 marks]

3.6.3 Solutions to 3.4 Example

(i)

$$\begin{aligned}
\text{LHS} &= \frac{\sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta}{\theta} \\
&\approx \frac{\theta + 2\theta + 3\theta + 4\theta}{\theta} \\
&= \frac{10\theta}{\theta} \\
&= 10 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

(ii) when $\theta = 0.1$,

$$\frac{\sin 0.1 + \sin 0.2 + \sin 0.3 + \sin 0.4}{0.1} = 9.834$$

$$\%age\ error = \frac{10 - 9.834}{9.834} \times 100 = 1.7\,\%$$

[3 marks]

3.6.4 Solutions to 3.5 Exercise

Answer 1

(i)

$$\begin{aligned}
\text{LHS} &= \frac{4\ \sin 3\theta\ -\ \tan 2\theta}{2\theta} \\
&\approx \frac{4\ \times\ 3\theta\ -\ 2\theta}{2\theta} \\
&= \frac{10\theta}{2\theta} \\
&= 5 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[2 marks]

(ii) when $\theta = 0.1$,

$$\frac{4\sin 0.3\ -\ \tan 0.2}{0.2}\ =\ 4.899$$

$$\%age\ error\ =\ \frac{5\ -\ 4.899}{4.899}\ \times\ 100\ =\ 2.1\ \%$$

[3 marks]

Answer 2

(i)

$$\begin{aligned}
\text{LHS} &= \frac{\cos 4\theta\ -\ 1}{\theta\ \sin 2\theta} \\
&\approx \frac{\left(1\ -\ \frac{(4\theta)^2}{2}\right)\ -\ 1}{2\ \theta^2} \\
&= \frac{1\ -\ 8\theta^2\ -\ 1}{2\theta^2} \\
&= -\frac{8\ \theta^2}{2\ \theta^2} \\
&= -4 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

(ii) when $\theta = 0.1$,

$$\frac{\cos 0.4\ -\ 1}{0.1\ \times\ \sin 0.2}\ =\ -3.973$$

$$\%age\ error\ =\ \frac{-\ 4\ +\ 3.973}{-\ 3.973}\ \times\ 100\ =\ 0.68\ \%$$

[3 marks]

Answer 3

$$\frac{1 - \cos 4\theta}{2\theta \sin 3\theta} \approx \frac{1 - \left(1 - \frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$$

$$= \frac{8\,\theta^2}{6\,\theta^2}$$

$$= \frac{4}{3}$$

[3 marks]

Answer 4

(i) LHS $= \cos 2\theta - \sin^2 3\theta$

$$\approx \left(1 - \frac{(2\theta)^2}{2}\right) - (3\theta)^2$$

$$= 1 - 2\,\theta^2 - 9\,\theta^2$$

$$= 1 - 11\,\theta^2$$

$$= \text{RHS} \qquad \square$$

[2 marks]

(ii) When θ is small, θ^2 is negligible in comparison with the number 1

[1 mark]

(iii) when $\theta = 0.1^c$

$$\sin^2 3\theta - \cos 2\theta = 0.8927343853$$

$$\%age\ error = \frac{1 - 0.8927343853}{0.8927343853} \times 100 = 12\ \%$$

[2 marks]

Answer 5

(i)

$$\text{LHS} = \frac{4\cos 3\theta - 2 + 5\sin\theta}{1 - \sin 2\theta}$$

$$\approx \frac{4\left(1 - \frac{(3\theta)^2}{2}\right) - 2 + 5\theta}{1 - 2\theta}$$

$$= \frac{4 - 18\,\theta^2 - 2 + 5\theta}{1 - 2\theta}$$

$$= \frac{2 + 5\theta - 18\,\theta^2}{1 - 2\theta}$$

$$= \frac{(2 + 9\theta)(1 - 2\theta)}{(1 - 2\theta)}$$

$$= 9\theta + 2$$

$$= \text{RHS} \qquad \square$$

[3 marks]

(ii) when θ = 0.2

$$\frac{4\,cos\,0.6\;-\;2\;+\;5\,sin\,0.2}{1\;-\;sin\,0.4}\;=\;3.758201847$$

$$\%age\;error\;=\;\frac{3.8\;-\;3.758201847}{3.758201847}\;\times\;100\;=\;1.11\;\%$$

[2 marks]

Answer 6

(i)

$$cos\,\theta\;+\;tan^2\,\theta$$

$$\approx\;\left(1\;-\;\frac{\theta^2}{2}\right)\;+\;\theta^2$$

$$=\;1\;+\;\frac{\theta^2}{2}$$

(ii) When θ is small and measured in radians the term in θ^2 can be ignored and so the required approximation is obtained.

i.e. $cos\,\theta\;+\;tan^2\,\theta\;\approx\;1$

[2, 1 marks]

Answer 7

(i)

$$5\,sin\,2\theta\;+\;tan\,3\theta\;=\;0.39$$

for small values of θ becomes.....

$$5\;\times\;2\theta\;+\;3\theta\;=\;0.39$$

$$13\theta\;=\;0.39$$

$$\theta\;=\;0.03^{\,c}$$

(ii) Use of the small angle approximations was valid for this solution as it is small relative to the period of the trigonometric waveform under consideration.

[2, 2 marks]

Answer 8

(i)

$$sin\,5\theta\;-\;cos\,2\theta\;+\;tan\,2\theta$$

$$\approx\;5\theta\;-\left(1\;-\;\frac{(2\theta)^2}{2}\right)\;+\;2\theta$$

$$=\;2\,\theta^2\;+\;7\theta\;-\;1$$

[3 marks]

(ii) Jason claim that the approximation will have value – 1 however, when θ = 0.1,

$$2 \times 0.1^2 + 7 \times 0.1 - 1 = -0.28$$

$\therefore$ Jason is incorrect

In general it's incorrect to ignore a term in θ, because θ then has to be so small for the approximation to be valid that it's no longer useful.

[1 mark]

The Edexcel A-Level Pure Mathematics textbook has this wrong on page 134 where the incorrect Example 21 inspired this question

Answer 9

(i)

$$Arc\ Length = r\,\theta$$

$$0.6 = r\,\theta$$

$$r = \frac{0.6}{\theta}$$

$$Area\ \Delta = \frac{1}{2}\,a\,b\,sin\,C$$

$$12 = \frac{1}{2}\,r^2\,sin\,\theta$$

$$24 = \left(\frac{0.6}{\theta}\right)^2 sin\,\theta$$

using the small angle approximation

$$24 = \frac{0.36}{\theta}$$

$$\therefore \theta = 0.015^c$$

[4 marks]

(ii) When θ is small we're effectively saying that the area of the triangle is roughly the same as the area of the sector, and that the area of the segment is negligible.

[1 mark]