

5.2 Answers

A-Level Pure Mathematics : Year 2 Applications of Trigonometry

Answer 1

- (i) 90° (ii) 150° (iii) 195°

[3 marks]

Answer 2

- (i) $\frac{\pi}{4}$ (ii) $\frac{\pi}{12}$ (iii) $\frac{11\pi}{6}$

[3 marks]

Answer 3

$$\begin{aligned}2 \cos \left(2x + \frac{\pi}{8} \right) &= \sqrt{2} \\ \cos \left(2x + \frac{\pi}{8} \right) &= \frac{\sqrt{2}}{2} \\ \left(2x + \frac{\pi}{8} \right) &= \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4} \\ 2x &= \frac{\pi}{8}, \frac{13\pi}{8}, \frac{17\pi}{8}, \frac{29\pi}{8} \\ x &= \frac{\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{29\pi}{16}\end{aligned}$$

[6 marks]

Answer 4

- (a) The student has used an angle measured in degrees in a formula that requires the angle to be in radians.
(b)

$$\begin{aligned}\text{Area of sector} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \times 5^2 \times 40 \times \frac{\pi}{180} \\ &= \frac{25\pi}{9} \\ &= 8.73 \text{ cm}^2\end{aligned}$$

[1, 2 marks]

Answer 5

(i) $\frac{1 - \cos 3\theta}{\theta \tan 2\theta} \approx \frac{1 - \left(1 - \frac{(3\theta)^2}{2} \right)}{\theta \times (2\theta)} = \frac{9\theta^2}{4\theta^2} = \frac{9}{4}$

- (ii) Exact value is 2.203320164

$$\begin{aligned}\% \text{ error} &= \frac{2.25 - 2.203320164}{2.203320164} \times 100 \\ &= 2.12 \%\end{aligned}$$

[4, 3 marks]

Answer 6

$$4 \sin x = \sec x$$

$$4 \sin x = \frac{1}{\cos x}$$

$$4 \sin x \cos x = 1$$

$$2 \sin x \cos x = \frac{1}{2}$$

$$\sin 2x = \frac{1}{2}$$

$$2x = \arcsin\left(\frac{1}{2}\right)$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$

[4 marks]**Answer 7**Use the cosine rule to get length BC ,

$$\begin{aligned} BC^2 &= 7.5^2 + 8.5^2 - 2 \times 7.5 \times 8.5 \times \cos(\pi - 1.2) \\ &= 174.7 \end{aligned}$$

$$\therefore BC = 13.217$$

Use arc length formula to get the length of the arc AB ,

$$\begin{aligned} \text{Arc length } AB &= 7.5 \times 1.2 \\ &= 9 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 13.217 + 9 + 7.5 + 8.5 \\ &= 38.2 \text{ cm} \quad (\text{one decimal place}) \end{aligned}$$

[5 marks]

Answer 8**(a)**

$$\begin{aligned}
5 \cos x - 3 \sin x &= R \cos (x + \alpha) \\
&= R \cos x \cos \alpha - R \sin x \sin \alpha \\
\therefore \tan \alpha &= \frac{R \sin \alpha}{R \cos \alpha} \\
&= \frac{3}{5} \\
\alpha &= \arctan \left(\frac{3}{5} \right) \\
&= 0.540 \text{ radians} \\
R &= \sqrt{3^2 + 5^2} \\
&= \sqrt{34} \\
&= 5.83
\end{aligned}$$

Thus,

$$5 \cos x - 3 \sin x = \sqrt{34} \cos (x + 0.540^c)$$

[4 marks]**(b)**

$$\begin{aligned}
5 \cos x - 3 \sin x &= 4 \\
\sqrt{34} \cos (x + 0.540) &= 4 \\
\cos (x + 0.540) &= 0.686 \\
x + 0.540 &= \arccos 0.686 \\
&= 0.815, 2\pi - 0.815 \\
x &= 0.275^c, 4.93^c
\end{aligned}$$

[5 marks]