6.2 Answers

A-Level Pure Mathematics : Year 2
Applications of Trigonometry

Answer 1

(i) $60°$ **(ii)** $84°$ **(iii)** $255°$

[3 marks]

Answer 2

(i) $\frac{\pi}{6}$ **(ii)** $\frac{5\pi}{3}$ **(iii)** $\frac{5\pi}{12}$

[3 marks]

Answer 3

$$2\sin\left(2x - \frac{\pi}{6}\right) = \sqrt{3}$$

$$\sin\left(2x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\left(2x - \frac{\pi}{6}\right) = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$$

$$2x = \frac{\pi}{2}, \frac{5\pi}{6}, \frac{5\pi}{2}, \frac{17\pi}{6}$$

$$x = \frac{\pi}{4}, \frac{5\pi}{12}, \frac{5\pi}{4}, \frac{17\pi}{12}$$

[5 marks]

Answer 4

(a) METHOD 1 $\quad LHS = 1 + 4\cos\theta + 3\cos^2\theta$

$$\approx 1 + 4\left(1 - \frac{\theta^2}{2}\right) + 3\left(1 - \frac{\theta^2}{2}\right)^2$$

$$= 1 + 4 - 2\theta^2 + 3\left(1 - \theta^2 + \frac{\theta^4}{4}\right)$$

$$= 5 - 2\theta^2 + 3 - 3\theta^2 + \frac{3}{4}\theta^4$$

$$= 8 - 5\theta^2 + \frac{3}{4}\theta^4$$

$\approx 8 - 5\theta^2$ for small values of θ

$= RHS \quad \square$

[3 marks]

METHOD 2 $\quad LHS = 1 + 4\cos\theta + 3\cos^2\theta$

$$= 1 + 4\cos\theta + 3(1 - \sin^2\theta)$$

$$= 1 + 4\left(1 - \frac{\theta^2}{2}\right) + 3 - 3\theta^2$$

$$= 1 + 4 - 2\theta^2 + 3 - 3\theta^2$$

$$= 8 - 5\theta^2$$

$= RHS \quad \square$

[3 marks]

(b) **(i)** Adele has used degrees where radians should be used.

(ii) $1 + 4\cos\left(5 \times \frac{\pi}{180}\right) + 3\cos^2\left(5 \times \frac{\pi}{180}\right) = 7.962$

$$8 - 5\theta^2 = 8 - 5 \times \left(5 \times \frac{\pi}{180}\right)^2 = 7.962$$

$\therefore 8 - 5\theta^2$ is a good approximation for $1 + 4\cos\theta + 3\cos^2\theta$ when $\theta = 5°$

[2 marks]

Answer 5

$$\begin{aligned} Area\ Sector &= \frac{1}{2} r^2 \theta \\ 135 &= \frac{1}{2} \times r^2 \times 4.8 \\ r^2 &= \frac{225}{4} \\ r &= \frac{15}{2} \end{aligned}$$

$$\begin{aligned} (Minor)\ Arc\ Length\ AB &= r\,\theta_{minor} \\ &= 7.5 \times (2\pi - 4.8) \\ &= 15\pi - 36 \text{ cm} \qquad \text{(It's about 11.1... cm)} \\ \therefore\ a &= 15,\ b = -36 \end{aligned}$$

[4 marks]

Answer 6

$$\begin{aligned} 4\cos x &= \csc x \\ 4\cos x &= \frac{1}{\sin x} \\ 4\sin x \cos x &= 1 \\ 2\sin x \cos x &= \frac{1}{2} \\ \sin 2x &= \frac{1}{2} \\ 2x &= \arcsin\left(\frac{1}{2}\right) \\ 2x &= \frac{\pi}{6}, \frac{5\pi}{6} \\ x &= \frac{\pi}{12}, \frac{5\pi}{12} \end{aligned}$$

[4 marks]

Answer 7

(a)

Use the sine rule to get angle *ACB*,

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\sin C = \frac{5 \times \sin 0.6}{4}$$

$$= 0.706$$

$$C = \arcsin 0.706$$

$$= 0.7835$$

Also 2.358 is a solution but is not needed as we're told $\angle ABC$ is the largest angle

$$\therefore \angle ABC = \pi - 0.6 - 0.7835$$

$$= 1.76, \quad \text{as expected}$$

[4 marks]

(b)

$$\text{Sector Area} = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \times 4^2 \times (\pi - 1.76)$$

$$= 11.1 \text{ cm}^2$$

$$\text{Area } \Delta ABC = \frac{1}{2} \times 5 \times 4 \times \sin 1.76$$

$$= 9.8$$

$$\therefore \text{ Area Emblem } = 20.9 \text{ cm}^2$$

[3 marks]

Answer 8

(a)

$$\begin{aligned} 5\cos x + 12\sin x &= R\cos(x-\alpha) \\ &= R\cos x\cos\alpha + R\sin x\sin\alpha \end{aligned}$$

$$\text{Now,}\quad \tan\alpha = \frac{R\sin\alpha}{R\cos\alpha}$$

$$= \frac{12}{5}$$

$$\alpha = \arctan\left(\frac{12}{5}\right)$$

$$= 1.176 \text{ radians}$$

$$R = \sqrt{12^2 + 5^2}$$

$$= 13$$

Thus,

$$5\cos x + 12\sin x = 13\cos(x - 1.176^c)$$

[4 marks]

(b)

$$5\cos x + 12\sin x = 6$$

$$13\cos(x - 1.176) = 6$$

$$\cos(x - 1.176) = 0.462$$

$$x - 1.176 = \arccos 0.462$$

$$= -1.091,\ 1.091$$

$$x = 0.085,\ 2.267$$

[5 marks]

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