2.3 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Binomial Expansion

2.3.1 Solution to 2.2 Making Use of Pascal's Triangle (Teaching Video)

To expand the brackets of $(1 + x)^5$ use Row 5 of Pascal's Triangle.
Note that Row 5 is actually the 6th row, but the 1st row is Row Zero.

1									
1	1								
1	2	1							
1	3	3	1						
1	4	6	4	1					
1	5	10	10	5	1				
1	6	15	20	15	6	1			
1	7	21	35	35	21	7	1		
1	8	28	56	70	56	28	8	1	
1	9	36	84	126	126	84	36	9	1

$$(1 + x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

[3 marks]

2.3.2 Solutions to 2.2 Exercise

Answer 1

$$(1+x)^8 = 1 + 8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x^8$$

[3 marks]

Answer 2

The result is always a cube number.

[6 marks]

Out of interest, here is an A-Level standard proof.
A more accessible version of this (for GCSE students) should be easy to formulate as it only involves triangular numbers, counting numbers and the number 1.

$$n^3 = \binom{n+1}{2}\binom{n-1}{1}\binom{n}{0} + \binom{n+1}{1}\binom{n}{2}\binom{n-1}{0} + \binom{n}{1}$$

Proof

$$\binom{n+1}{2}\binom{n-1}{1}\binom{n}{0} = \frac{(n+1)n}{2} \times (n-1) \times 1$$
$$= \frac{(n+1)n(n-1)}{2}$$

$$\binom{n+1}{1}\binom{n}{2}\binom{n-1}{0} = (n+1) \times \frac{n(n-1)}{2} \times 1$$
$$= \frac{(n+1)n(n-1)}{2}$$

The sum is thus,

$$\binom{n+1}{2}\binom{n-1}{1}\binom{n}{0} + \binom{n+1}{1}\binom{n}{2}\binom{n-1}{0} + \binom{n}{1}$$
$$= \frac{(n+1)n(n-1)}{2} + \frac{(n+1)n(n-1)}{2} + n$$
$$= (n+1)n(n-1) + n$$
$$= \left(n^2 - 1\right)n + n$$
$$= n^3 - n + n$$
$$= n^3 \qquad \square$$

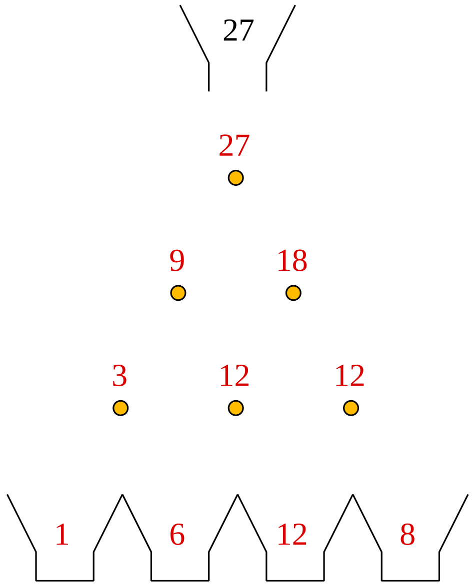
Answer 3

$$(1+x)^{10}$$
$$= 1 + 10x + 45x^2 + 120x^3 + 210x^4 + 252x^5 + 210x^6 + 120x^7 + 45x^8 + 10x^9 + x^{10}$$

[4 marks]

Answer 4

(i) and **(ii)**



[4 marks]

Answer 5

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

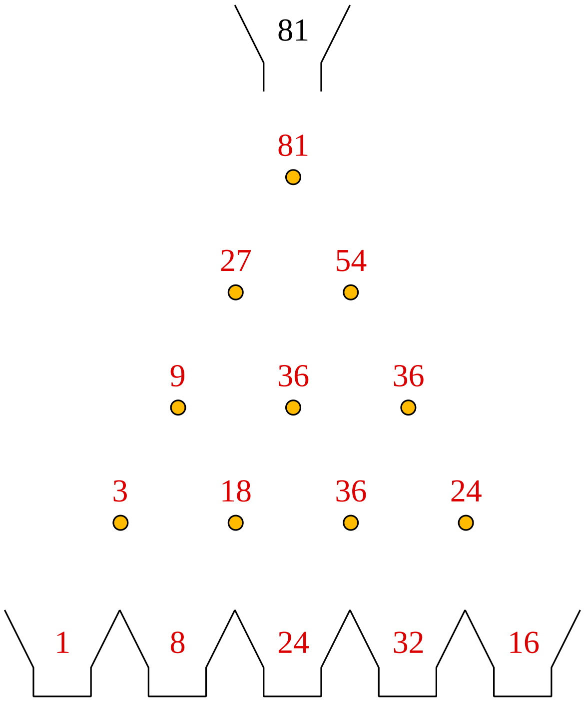
1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

[3 marks]

Answer 6



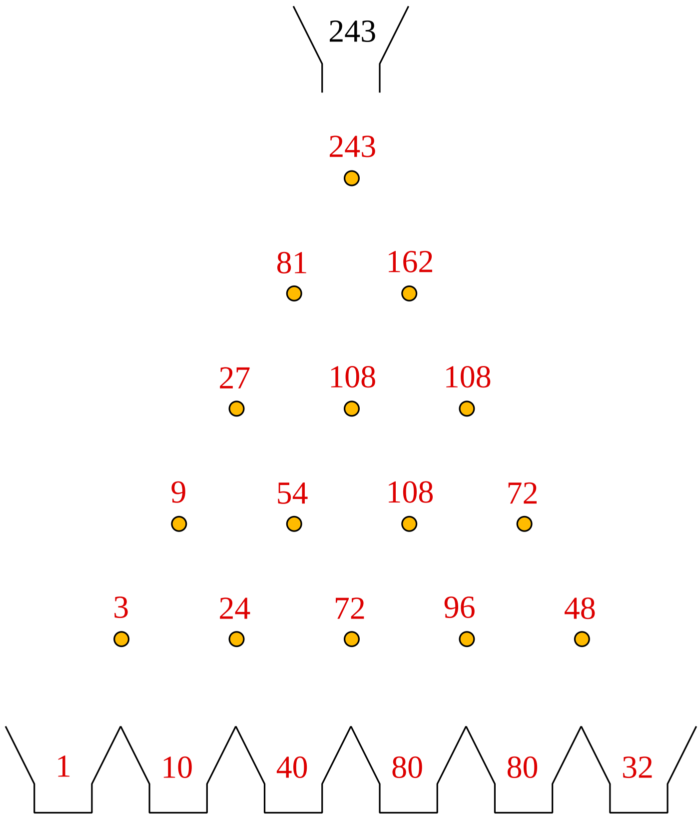
[6 marks]

Answer 7

$$(1 - x)^7 = 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$$

[3 marks]

Answer 8



[11 marks]

Answer 9

Starting Number of Balls	Distribution of Balls in Baskets
27	1 6 12 8
81	1 8 24 32 16
243	1 10 40 80 80 32

[4 marks]

Answer 10

$$(1 + 2x)^4 = 1 + 4(2x) + 6(2x)^2 + 4(2x)^3 + (2x)^4$$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

And there is the "1 8 24 32 16"

[3 marks]

Answer 11

$$(1 + 2x)^5 = 1 + 5(2x) + 10(2x)^2 + 10(2x)^3 + 5(2x)^4 + (2x)^5$$

$$= 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$$

And there is the "1 10 40 80 80 32"

[3 marks]