3.3 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Binomial Expansion

3.3.1 Solution to 3.1 Pascal To The Max (Teaching Video)

From Pascal's Triangle,

$$(1 + x)^3 = 1 + 3x + 3x^2 + x^3$$

with the x replaced with $(-4x)$ is obtained,

$$(1 - 4x)^3 = 1 + 3(-4x) + 3(-4x)^2 + (-4x)^3$$
$$= 1 - 12x + 48x^2 - 64x^3$$

[4 marks]

3.3.2 Solutions to 3.2 Exercise

Answer 1
From Pascal's Triangle,

$$(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$$

with the x replaced with $(3x)$ is obtained,

$$(1 + 3x)^4 = 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$$
$$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$$

[4 marks]

Answer 2
From Pascal's Triangle,

$$(1 + x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

with the x replaced with $(-2x)$ is obtained,

$$(1 - 2x)^5 = 1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$$
$$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$$

[4 marks]

Answer 3
The result is always a square number.

[4 marks]

Out Of Interest : A Proof

This could be proven using the formula for Triangular Numbers.

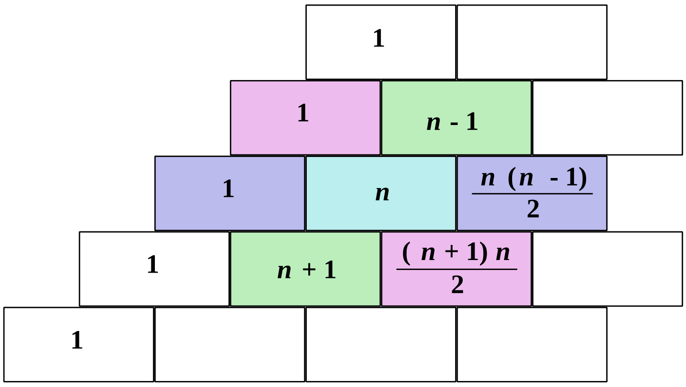
$$T_{ROW} = \frac{(Row)(Row - 1)}{2}$$

In Row 6, for example,

$$T_6 = \frac{6 \times 5}{2} = 15$$

Which is correct : 1 6 15 20 15 6 1

In general



$$\left(\frac{(n+1)n}{2} - 1\right) + \left((n+1) - (n-1)\right) + \left(\frac{n(n-1)}{2} - 1\right)$$

$$= \frac{n^2 + n - 2}{2} + \frac{4}{2} + \frac{n^2 - n - 2}{2}$$

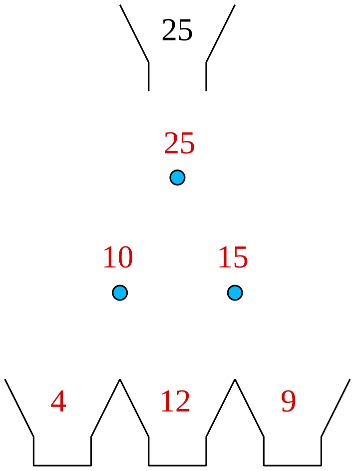
$$= \frac{n^2 + n - 2 + 4 + n^2 - n - 2}{2}$$

$$= \frac{2n^2}{2}$$

$$= n^2 \qquad \square$$

Answer 4

(i) and **(ii)**



[4 marks]

Answer 5

From Pascal's Triangle,

$$(1 + x)^7 = 1 + 7x + 21x^2 + 35x^3 + \ldots$$

with the x replaced with $(6x)$ is obtained,

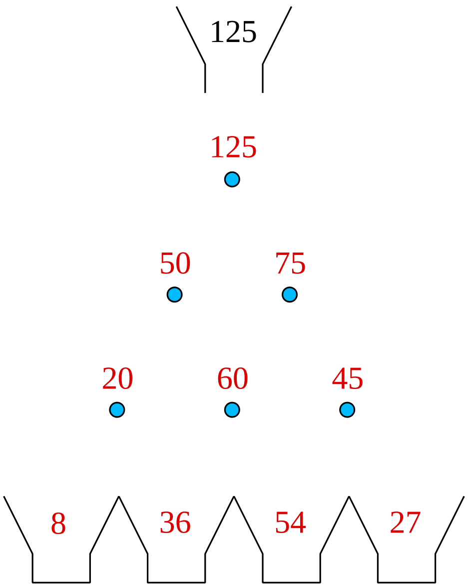
$$(1 + 6x)^7 = 1 + 7(6x) + 21(6x)^2 + 35(6x)^3 + \ldots$$

$$= 1 + 42x + 756x^2 + 7560x^3 + \ldots$$

[4 marks]

Answer 6

(i) and **(ii)**



[6 marks]

Answer 7

The sign will alternate,

$$1 - 12x + 66x^2 - 220x^3 + \ldots$$

[3 marks]

Answer 8

From Pascal's Triangle,

$$(1 + x)^8 = 1 + 8x + 28x^2 + 56x^3 + \ldots$$

with the x replaced with $\left(-\frac{x}{2}\right)$ is obtained,

$$\left(1 - \frac{x}{2}\right)^8 = 1 + 8\left(-\frac{x}{2}\right) + 28\left(-\frac{x}{2}\right)^2 + 56\left(-\frac{x}{2}\right)^3 + \ldots$$

$$= 1 - 4x + 7x^2 - 7x^3 + \ldots$$

[4 marks]

Answer 9

$$(x + y)^7$$

$$= x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

[3 marks]

Answer 10

$$(2 + 3x)^3 = 2^3 + 3(2)^2(3x) + 3(2)(3x)^2 + (3x)^3$$

$$= 8 + 36x + 54x^2 + 27x^3$$

[4 marks]