

4.4 Answers

Additional Mathematics A-Level Pure Mathematics : Year 1 **Binomial Expansion**

4.4.1 Solution to 4.3 Example (Teaching Video)

$$\begin{aligned}(3 - 2x)^4 = & 1 \times (3)^4 \times (-2x)^0 \\ & + 4 \times (3)^3 \times (-2x)^1 \\ & + 6 \times (3)^2 \times (-2x)^2 \\ & + 4 \times (3)^1 \times (-2x)^3 \\ & + 1 \times (3)^0 \times (-2x)^4\end{aligned}$$

So,

$$(3 - 2x)^4 = 81 - 216x + 216x^2 - 96x^3 + 16x^4$$

[4 marks]

2.5.3 Solutions to 2.4 Exercise

Answer 1

$$\begin{aligned}(5 + 3x)^3 = & 1 \times (5)^3 \times (3x)^0 \\ & + 3 \times (5)^2 \times (3x)^1 \\ & + 3 \times (5)^1 \times (3x)^2 \\ & + 1 \times (5)^0 \times (3x)^3\end{aligned}$$

Simplifying,

$$(5 + 3x)^3 = 125 + 225x + 135x^2 + 27x^3$$

[4 marks]

Answer 2

$$\begin{aligned}(2 - 5x)^5 = & 1 \times (2)^5 \times (-5x)^0 \\ & + 5 \times (2)^4 \times (-5x)^1 \\ & + 10 \times (2)^3 \times (-5x)^2 \\ & + 10 \times (2)^2 \times (-5x)^3 \\ & + 5 \times (2)^1 \times (-5x)^4 \\ & + 1 \times (2)^0 \times (-5x)^5\end{aligned}$$

Simplifying,

$$(2 - 5x)^5 = 32 - 400x + 2000x^2 - 5000x^3 + 6250x^4 - 3125x^5$$

[5 marks]

Answer 3

$$\begin{aligned}
 (4 + x^2)^3 = & \quad 1 \quad \times \quad (4)^3 \quad \times \quad (x^2)^0 \\
 & + \quad 3 \quad \times \quad (4)^2 \quad \times \quad (x^2)^1 \\
 & + \quad 3 \quad \times \quad (4)^1 \quad \times \quad (x^2)^2 \\
 & + \quad 1 \quad \times \quad (4)^0 \quad \times \quad (x^2)^3
 \end{aligned}$$

Simplifying,

$$(4 + x^2)^3 = 64 + 48x^2 + 12x^4 + x^6$$

[4 marks]**Answer 4****(i)**

$$\begin{aligned}
 \left(x + \frac{1}{x}\right)^4 = & \quad 1 \quad \times \quad (x)^4 \quad \times \quad \left(\frac{1}{x}\right)^0 \\
 & + \quad 4 \quad \times \quad (x)^3 \quad \times \quad \left(\frac{1}{x}\right)^1 \\
 & + \quad 6 \quad \times \quad (x)^2 \quad \times \quad \left(\frac{1}{x}\right)^2 \\
 & + \quad 4 \quad \times \quad (x)^1 \quad \times \quad \left(\frac{1}{x}\right)^3 \\
 & + \quad 1 \quad \times \quad (x)^0 \quad \times \quad \left(\frac{1}{x}\right)^4
 \end{aligned}$$

Simplifying,

$$\left(x + \frac{1}{x}\right)^4 = x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$$

[4 marks]**(ii)** The term which is independent of x is the 6**[1 mark]****Answer 5**

${}^4C_0 = 1$

${}^4C_1 = 4$

${}^4C_2 = 6$

${}^4C_3 = 4$

${}^4C_4 = 1$

[2 marks]

Answer 6

$${}^{10}C_0 = 1 \qquad {}^{10}C_1 = 10 \qquad {}^{10}C_2 = 45 \qquad {}^{10}C_3 = 120$$

$$\begin{aligned} \left(2 + \frac{x}{4}\right)^{10} = & 1 \times (2)^{10} \times \left(\frac{x}{4}\right)^0 \\ & + 10 \times (2)^9 \times \left(\frac{x}{4}\right)^1 \\ & + 45 \times (2)^8 \times \left(\frac{x}{4}\right)^2 \\ & + 120 \times (2)^7 \times \left(\frac{x}{4}\right)^3 \\ & + \dots \end{aligned}$$

Simplifying,

$$\left(2 + \frac{x}{4}\right)^4 = 1024 + 1280x + 720x^2 + 240x^3 + \dots$$

[5 marks]**Answer 7**

$$\begin{aligned} \left(2x - \frac{1}{16x}\right)^8 = & 1 \times (2x)^8 \times \left(-\frac{1}{16x}\right)^0 \\ & + 8 \times (2x)^7 \times \left(-\frac{1}{16x}\right)^1 \\ & + 28 \times (2x)^6 \times \left(-\frac{1}{16x}\right)^2 \\ & + \dots \end{aligned}$$

Simplifying,

$$\left(2x - \frac{1}{16x}\right)^8 = 256x^8 - 64x^6 + 7x^4 + \dots$$

[5 marks]