5.2 Answers to 5.1 Homework

Additional Mathematics
A-Level Pure Mathematics : Year 1
Binomial Expansion

Answer 1

(i)

$^{15}C_3 = 455$ $^{15}C_4 = 1365$ $^{15}C_5 = 3003$

[3 marks]

(ii) $(1 + x)^{15} = 1 + 15x + 105x^2 + 455x^3 + 1365x^4 + 3003x^5$

[2 marks]

Answer 2

$$\begin{aligned}(1 + 6x)^4 = \quad & 1 \times (1)^4 \times (6x)^0 \\ + \quad & 4 \times (1)^3 \times (6x)^1 \\ + \quad & 6 \times (1)^2 \times (6x)^2 \\ + \quad & 4 \times (1)^1 \times (6x)^3 \\ + \quad & 1 \times (1)^0 \times (6x)^4\end{aligned}$$

Simplifying,

$$(1 + 6x)^4 = 1 + 24x + 216x^2 + 864x^3 + 1296x^4$$

[4 marks]

Answer 3

$$\begin{aligned}\left(1 + \frac{x}{2}\right)^{10} = \; & 1 \times (1)^{10} \times \left(\frac{x}{2}\right)^0 \\ & + 10 \times (1)^9 \times \left(\frac{x}{2}\right)^1 \\ & + 45 \times (1)^8 \times \left(\frac{x}{2}\right)^2 \\ & + 120 \times (1)^7 \times \left(\frac{x}{2}\right)^3 \\ & + \ldots\end{aligned}$$

Simplifying,

$$\left(1 + \frac{x}{2}\right)^{10} = 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \ldots$$

[4 marks]

Answer 4

$$(3-x)^6 = 1 \times (3)^6 \times (-x)^0 + 6 \times (3)^5 \times (-x)^1 + 15 \times (3)^4 \times (-x)^2 + \ldots$$

Simplifying,

$$(3-x)^6 = 729 - 1458x + 1215x^2 - \ldots$$

[4 marks]

Answer 5

(a)

$$\left(3-\frac{ax}{2}\right)^5 = 1 \times (3)^5 \times \left(-\frac{ax}{2}\right)^0 + 5 \times (3)^4 \times \left(-\frac{ax}{2}\right)^1 + 10 \times (3)^3 \times \left(-\frac{ax}{2}\right)^2 + 10 \times (3)^2 \times \left(-\frac{ax}{2}\right)^3 + \ldots$$

Simplifying,

$$\left(3-\frac{ax}{2}\right)^5 = 243 - \frac{405}{2}ax + \frac{135}{2}a^2x^2 - \frac{45}{4}a^3x^3 + \ldots$$

[4 marks]

(b)

$$-\frac{45}{4}a^3 = -\frac{405}{2}a$$

Multiply through by (-4)

$$45a^3 = 810a$$

$$a^2 = 18 \quad \text{OK to divide by } a \text{ as“}a \text{ is a positive constant”}$$

$$a = 3\sqrt{2}$$

[3 marks]