

8.3 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Binomial Expansion

Answer 1

$$\begin{aligned}(5x - x^2)^3 &= 1 \times (5x)^3 \times (-x^2)^0 \\ &\quad + 3 \times (5x)^2 \times (-x^2)^1 \\ &\quad + 3 \times (5x)^1 \times (-x^2)^2 \\ &\quad + 1 \times (5x)^0 \times (-x^2)^3\end{aligned}$$

So,

$$(5x - x^2)^3 = 125x^3 - 75x^4 + 15x^5 - x^6$$

[5 marks]

Answer 2

(i) 720 (ii) 3 628 800 (iii) 6 227 020 800

[3 marks]

Answer 3

70! is a likely answer

(Biggest that works on CASIO fx-991EX ClassWiz is $69! = 1.7 \times 10^{98}$)

[1 mark]

Answer 4

$$\begin{aligned}\frac{5000!}{4999!} &= \frac{5000 \times 4999 \times 4998 \times \dots \times 2 \times 1}{4999!} \\ &= \frac{5000 \times 4999!}{4999!} \\ &= 5000\end{aligned}$$

[2 marks]

Answer 5

(i) 100 (ii) 999 900 (iii) 8 242 405 980

[1, 2, 3 marks]

Answer 6

$$\begin{aligned}\frac{(n+4)!}{(n+1)!} &= \frac{(n+4)(n+3)(n+2)(n+1)!}{(n+1)!} \\ &= (n+4)(n+3)(n+2)\end{aligned}$$

[3 marks]

Answer 7

$$\begin{aligned}\frac{(n+1)!}{(n-1)!} &= \frac{(n+1)(n)(n-1)!}{(n-1)!} \\ &= (n+1)n\end{aligned}$$

[3 marks]

Answer 8

$$\begin{aligned}
{}^nC_2 &= \frac{n!}{2! (n-2)!} \\
&= \frac{n (n-1) (n-2)!}{2! (n-2)!} \\
&= \frac{n (n-1)}{2}
\end{aligned}$$

This is a formula for triangular numbers so the above is a proof that column 2 of Pascal's triangle will contain the triangular numbers.

[4 marks]

Answer 9

$$\begin{aligned}
(4 + 5x)^3 &= 1 \times (4)^3 \times (5x)^0 \\
&+ 3 \times (4)^2 \times (5x)^1 \\
&+ 3 \times (4)^1 \times (5x)^2 \\
&+ 1 \times (4)^0 \times (5x)^3
\end{aligned}$$

$$\text{So } (4 + 5x)^3 = 64 + 240x + 300x^2 + 125x^3$$

$$\begin{aligned}
(2 + x)(4 + 5x)^3 &= (2 + x)(64 + 240x + 300x^2 + 125x^3) \\
&= 128 + 480x + 600x^2 + 250x^3 \\
&\quad + 64x + 240x^2 + 300x^3 + 125x^4 \\
&= 128 + 544x + 840x^2 + 550x^3 + 125x^4
\end{aligned}$$

[8 marks]

Answer 10

$$\begin{aligned}
(2x + a)^6 &= 1 \times (2x)^6 \times (a)^0 \\
&+ 6 \times (2x)^5 \times (a)^1 \\
&+ 15 \times (2x)^4 \times (a)^2 \quad \Leftarrow \Leftarrow \Leftarrow \\
&+ \dots
\end{aligned}$$

$$\therefore 240 a^2 x^4 = 60 x^4$$

$$a^2 = \frac{1}{4}$$

$$a = \pm \frac{1}{2}$$

[5 marks]