

9.4 Answers

Additional Mathematics A-Level Pure Mathematics : Year 1 **Binomial Expansion**

9.4.1 Solution to the 9.2 Example (Teaching Video)

$$\begin{aligned}(a + 3x)^3 &= 1 \times (a)^3 \times (3x)^0 \\ &\quad + 3 \times (a)^2 \times (3x)^1 \\ &\quad + 3 \times (a)^1 \times (3x)^2 \\ &\quad + 1 \times (a)^0 \times (3x)^3 \\ &= a^3 + 9a^2x + 27ax^2 + 27x^3\end{aligned}$$

The question tells us that; $9a^2 = 27$

$$a^2 = 3$$

$$a = \pm\sqrt{3}$$

[5 marks]

9.4.2 Solutions to the 9.3 Exercise

Answer 1

$$\begin{aligned}(a + 5x)^4 &= 1 \times (a)^4 \times (5x)^0 \\ &\quad + 4 \times (a)^3 \times (5x)^1 \\ &\quad + 6 \times (a)^2 \times (5x)^2 \\ &\quad + 4 \times (a)^1 \times (5x)^3 \\ &\quad + 1 \times (a)^0 \times (5x)^4 \\ &= a^4 + 20a^3x + 150a^2x^2 + 500ax^3 + 625x^4\end{aligned}$$

The question tells us that; $20a^3 = 500a$

$$a^2 = 25$$

$$a = \pm 5$$

[5 marks]

Answer 2

$$\begin{aligned}
(2 + ax)^3 &= 1 \times (2)^3 \times (ax)^0 \\
&\quad + 3 \times (2)^2 \times (ax)^1 \\
&\quad + 3 \times (2)^1 \times (ax)^2 \\
&\quad + 1 \times (2)^0 \times (ax)^3 \\
&= 8 + 12ax + 6a^2x^2 + a^3x^3
\end{aligned}$$

The question tells us that; $a^3 = 12a$

$$a^2 = 12$$

$$a = \pm\sqrt{12}$$

$$a = \pm 2\sqrt{3}$$

[5 marks]

Answer 3

$$\begin{aligned}
\text{(a)} \quad (1 + kx)^6 &= 1 \times (1)^6 \times (kx)^0 \\
&\quad + 6 \times (1)^5 \times (kx)^1 \\
&\quad + 15 \times (1)^4 \times (kx)^2 \\
&\quad + 20 \times (1)^3 \times (kx)^3 \\
&\quad + \dots
\end{aligned}$$

$$= 1 + 6kx + 15k^2x^2 + 20k^3x^3 + \dots$$

[5 marks]

(b)

$$15k^2 = 6k$$

$$15k = 6 \quad \text{no division by zero as told } k \neq 0$$

$$k = \frac{2}{5}$$

[2 marks]

(c)

$$20k^3 = 20 \times \left(\frac{2}{5}\right)^3$$

$$= \frac{32}{25}$$

[1 mark]

Answer 4

$$\begin{aligned}
 \text{(a)} \quad (2 + kx)^7 &= 1 \times (2)^7 \times (kx)^0 \\
 &+ 7 \times (2)^6 \times (kx)^1 \\
 &+ 21 \times (2)^5 \times (kx)^2 \\
 &+ 35 \times (2)^4 \times (kx)^3 \\
 &+ \dots \\
 &= 128 + 448kx + 672k^2x^2 + 560k^3x^3 + \dots
 \end{aligned}$$

(only the first three terms are required)

[5 marks]

$$\begin{aligned}
 \text{(b)} \quad 672k^2 &= 6 \times 448k \\
 &\text{divide both sides by 672} \\
 k^2 &= 4k \\
 k^2 - 4k &= 0 \\
 k(k - 4) &= 0 \\
 \text{Either } k &= 0 \quad \text{or} \quad k = 4
 \end{aligned}$$

(I suspect the $k = 0$ answer was not intended by the question setter)

[2 marks]

Answer 5

$$\begin{aligned}
 \text{(a)} \quad (5 + px)^{30} &= 1 \times (5)^{30} \times (px)^0 \\
 &+ 30 \times (5)^{29} \times (px)^1 \\
 &+ 435 \times (5)^{28} \times (px)^2 \\
 &+ 4060 \times (5)^{27} \times (px)^3 \\
 &+ \dots \\
 &= 5^{30} + 5^{29} \times 30px + 5^{28} \times 435p^2x^2 + 5^{27} \times 4060p^3x^3 + \dots
 \end{aligned}$$

(only the first three terms required)

[2 marks]

$$\text{(b)} \quad 5^{28} \times 435p^2 = 29 \times 5^{29} \times 30p$$

Divide through by 5^{29} and also by p as we're told $p \neq 0$

$$87p = 29 \times 30$$

$$p = 10$$

[4 marks]

Answer 6**(a)**

$$\begin{aligned}
\left(2 - \frac{x}{16}\right)^9 &= 1 \times (2)^9 \times \left(-\frac{x}{16}\right)^0 \\
&+ 9 \times (2)^8 \times \left(-\frac{x}{16}\right)^1 \\
&+ 36 \times (2)^7 \times \left(-\frac{x}{16}\right)^2 \\
&+ \dots \\
&= 512 - 144x + 18x^2 + \dots
\end{aligned}$$

$$\begin{aligned}
(a + bx) \left(2 - \frac{x}{16}\right)^9 &= (a + bx) (512 - 144x + 18x^2 + \dots) \\
&= 512a - 144ax + 512bx + \dots
\end{aligned}$$

[5 marks]**(b)**

$$512a = 128$$

$$a = \frac{1}{4}$$

[2 marks]**(c)**

$$-144a + 512b = 36$$

$$-36 + 512b = 36$$

$$b = \frac{72}{512}$$

$$b = \frac{9}{64}$$

[2 marks]